$$
\begin{aligned}
e^{a+bi} &= e^{a}(\cos b + i\sin b)\\
\cosh(x) &= \tfrac{1}{2}\left(e^{x} + e^{-x}\right)\\
\sinh(x) &= \tfrac{1}{2}\left(e^{x} - e^{-x}\right)\\
\cos(x) &= \cosh(ix)\\
\sin(x) &= -i\sinh(ix)\\
\Gamma(x) &= \int_0^\infty e^{-t}\, t^{x-1}\, dt\\
\Gamma(n) &= (n-1)!
\end{aligned}
$$

$$
\begin{aligned}
\delta(-t) &= \delta(t)\\
\delta(at) &= \frac{1}{|a|}\delta(t)\\
t\frac{d}{dt}\delta(t) &= -\delta(t)\\
\frac{d^n}{dt^n}\delta(t-x) &= (-1)^n\delta(t-x)\frac{d^n}{dt^n}\\
\frac{d}{dt}\theta(t-x) &= \delta(t-x)\\
\theta(t) &= \begin{cases} 0, & \text{kun } t<0\\ 1, & \text{kun } t\geq 0\end{cases}
\end{aligned}
$$

$$
\begin{aligned}
e^{x} &= \sum_{n=0}^{\infty}\frac{x^n}{n!}\\
\sin(x) &= \sum_{n=0}^{\infty}\frac{(-1)^n\, x^{2n+1}}{(2n+1)!}\\
\cos(x) &= \sum_{n=0}^{\infty}\frac{(-1)^n\, x^{2n}}{(2n)!}\\
\ln(1+x) &= \sum_{n=0}^{\infty}\frac{(-1)^{n+1}\, x^n}{n}, \quad |x|<1\\
\frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n, \quad |x|<1
\end{aligned}
$$

$$
\begin{aligned}
&y''(x) + \kappa^2 y(x) = 0\\
&\qquad\Longrightarrow y(x) = A\sin(\kappa x) + B\cos(\kappa x)\\
&y''(x) - \kappa^2 y(x) = 0\\
&\qquad\Longrightarrow y(x) = Ae^{\kappa x} + Be^{-\kappa x}
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos(n\omega x)\\
&\quad + \sum_{n=1}^{\infty} b_n\sin(n\omega x)\\
F(x) &= \sum_{n=-\infty}^{\infty} c_n e^{in\omega x}\\
a_n &= \tfrac{2}{P}\int_a^{a+P} f(x)\cos(n\omega x)\, dx\\
b_n &= \tfrac{2}{P}\int_a^{a+P} f(x)\sin(n\omega x)\, dx\\
c_n &= \tfrac{1}{P}\int_a^{a+P} f(x)e^{-in\omega x}\, dx\\
\omega &= \frac{2\pi}{P}\\
\tfrac{1}{P}\int_a^{a+P} f(x)^2\, dx &= \tfrac{1}{4}a_0^2 + \tfrac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}(f)(\alpha) &= \tfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-i\alpha x}\, dx\\
\mathcal{F}^{-1}(g)(x) &= \tfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(\alpha)e^{i\alpha x}\, d\alpha\\
\int_{-\infty}^{\infty} g_1(\alpha)g_2(-\alpha)\, d\alpha &= \int_{-\infty}^{\infty} f_1(x)f_2(x)\, dx\\
\mathcal{L}(f)(\alpha) &= \int_0^{\infty} f(x)e^{-\alpha x}\, dx\\
(f*g)(x) &= \int_0^{x} f(t)g(x-t)\, dt\\
\mathcal{L}(y')(\alpha) &= -y(0) + \alpha\mathcal{L}(y)(\alpha)
\end{aligned}
$$

Katso myös toinen puoli!

Taulukko 1. Hyödyllisiä Laplacen muunnoksia.

$f(x)$	$\mathcal{L}(f)(\alpha)$	Pätevyysalue
1	$1/\alpha$	$\mathrm{Re}(\alpha) > 0$
e^{-ax}	$\frac{1}{\alpha+a}$	$\mathrm{Re}(\alpha+a) > 0$
$\sin(ax)$	$\frac{a}{\alpha^2+a^2}$	$\mathrm{Re}(\alpha) > \lvert\mathrm{Im}(a)\rvert$
$\cos(ax)$	$\frac{\alpha}{\alpha^2+a^2}$	$\mathrm{Re}(\alpha) > \lvert\mathrm{Im}(a)\rvert$
x^k	$\frac{\Gamma(k+1)}{\alpha^{k+1}}$	$\mathrm{Re}(\alpha) > 0,\ k > -1$
$x^k\mathrm{e}^{-ax}$	$\frac{\Gamma(k+1)}{(\alpha+a)^{k+1}}$	$\mathrm{Re}(\alpha+a) > 0,\ k > -1$
$\frac{\mathrm{e}^{-ax}-\mathrm{e}^{-bx}}{b-a}$	$\frac{1}{(\alpha+a)(\alpha+b)}$	$\mathrm{Re}(\alpha+a) > 0,\ \mathrm{Re}(\alpha+b) > 0$
$\frac{a\mathrm{e}^{-ax}-b\mathrm{e}^{-bx}}{a-b}$	$\frac{\alpha}{(\alpha+a)(\alpha+b)}$	$\mathrm{Re}(\alpha+a) > 0,\ \mathrm{Re}(\alpha+b) > 0$
$\sinh(ax)$	$\frac{a}{\alpha^2-a^2}$	$\mathrm{Re}(\alpha) > \lvert\mathrm{Re}(a)\rvert$
$\cosh(ax)$	$\frac{\alpha}{\alpha^2-a^2}$	$\mathrm{Re}(\alpha) > \lvert\mathrm{Re}(a)\rvert$
$x\sin(ax)$	$\frac{2a\alpha}{(\alpha^2+a^2)^2}$	$\mathrm{Re}(\alpha) > \lvert\mathrm{Im}(a)\rvert$
$x\cos(ax)$	$\frac{\alpha^2-a^2}{(\alpha^2+a^2)^2}$	$\mathrm{Re}(\alpha) > \lvert\mathrm{Im}(a)\rvert$
$\mathrm{e}^{-ax}\sin(bx)$	$\frac{b}{(\alpha+a)^2+b^2}$	$\mathrm{Re}(\alpha+a) > \lvert\mathrm{Im}(b)\rvert$
$\mathrm{e}^{-ax}\cos(bx)$	$\frac{\alpha+a}{(\alpha+a)^2+b^2}$	$\mathrm{Re}(\alpha+a) > \lvert\mathrm{Im}(b)\rvert$
$1-\cos(ax)$	$\frac{a^2}{\alpha(\alpha^2+a^2)}$	$\mathrm{Re}(\alpha) > \lvert\mathrm{Im}(a)\rvert$
$ax-\sin(ax)$	$\frac{a^3}{\alpha^2(\alpha^2+a^2)}$	$\mathrm{Re}(\alpha) > \lvert\mathrm{Im}(a)\rvert$
$\sin(ax)-ax\cos(ax)$	$\frac{2a^3}{(\alpha^2+a^2)^2}$	$\mathrm{Re}(\alpha) > \lvert\mathrm{Im}(a)\rvert$
$\mathrm{e}^{-ax}(1-ax)$	$\frac{\alpha}{(\alpha+a)^2}$	$\mathrm{Re}(\alpha+a) > 0$
$\frac{\sin(ax)}{x}$	$\arctan\left(\frac{a}{\alpha}\right)$	$\mathrm{Re}(\alpha) > \lvert\mathrm{Im}(a)\rvert$
$\frac{\sin(ax)\cos(bx)}{x}$	$\frac{1}{2}\left\{\arctan\left(\frac{a+b}{\alpha}\right)+\arctan\left(\frac{a-b}{\alpha}\right)\right\}$	$\mathrm{Re}(\alpha) > 0,\ a, b > 0$
$\frac{\mathrm{e}^{-ax}-\mathrm{e}^{-bx}}{x}$	$\ln\left(\frac{\alpha+b}{\alpha+a}\right)$	$\mathrm{Re}(\alpha+a) > 0,\ \mathrm{Re}(\alpha+b) > 0$
$1-\mathrm{erf}\left(\frac{a}{2\sqrt{x}}\right)$	$\frac{1}{\alpha}\mathrm{e}^{-a\sqrt{\alpha}}$	$\mathrm{Re}(\alpha) > 0,\ a > 0$
$J_0(ax)$	$\frac{1}{\sqrt{\alpha^2+a^2}}$	$\mathrm{Re}(\alpha) > \lvert\mathrm{Im}(a)\rvert$

Katso myös toinen puoli!