**FYSA220 Sähköoppi** — **exam**, **Pe 21.05.2010**

**Notice: Solve 6 problems out of 7**

**1.** Two persons (mass about 70 kg) both have a charge of 400 C. This means that approximately 1 molecule out of million has been charged.

a) Calculate the electric field E produced by person A at the radius of 1000 km. (3 p)

b) Calculate the potential $\phi$ at the same radius. (3 p)

c) Define the work done when bringing up the person B from infinity to the point where the distance to person A is 1000 km. (4 p)

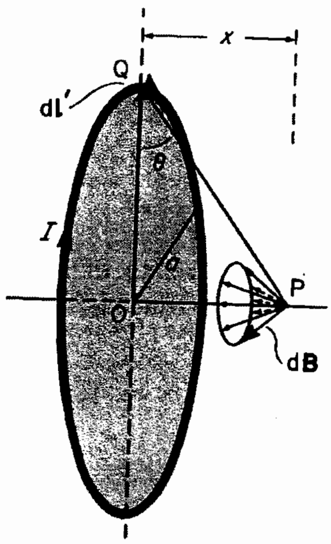
**2.** Starting from Biot-Savart equation

$$\overline{B}(\overline{r}) = \frac{\mu_0 I}{4\pi}\oint_s \frac{d\overline{l} \times (\overline{r} - \overline{r}')}{|\overline{r} - \overline{r}'|^3})$$

show that the magnetic field B produced by the current loop on the axis is

$$B = \frac{\mu_0 I}{2}\frac{a^2}{(x^2 + a^2)^{3/2}},$$

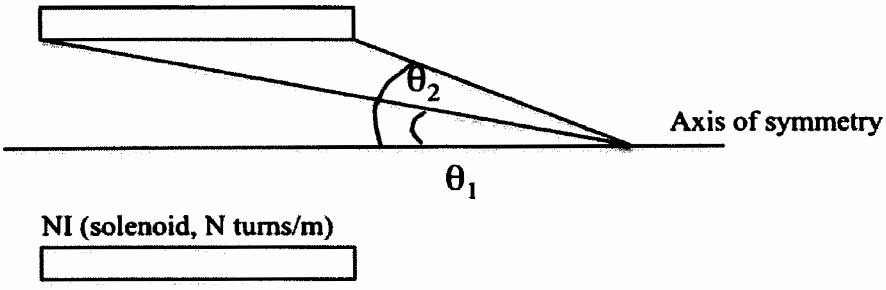
where $x$ is the distance from the loop and $a$ the radius of the loop (see fig.).



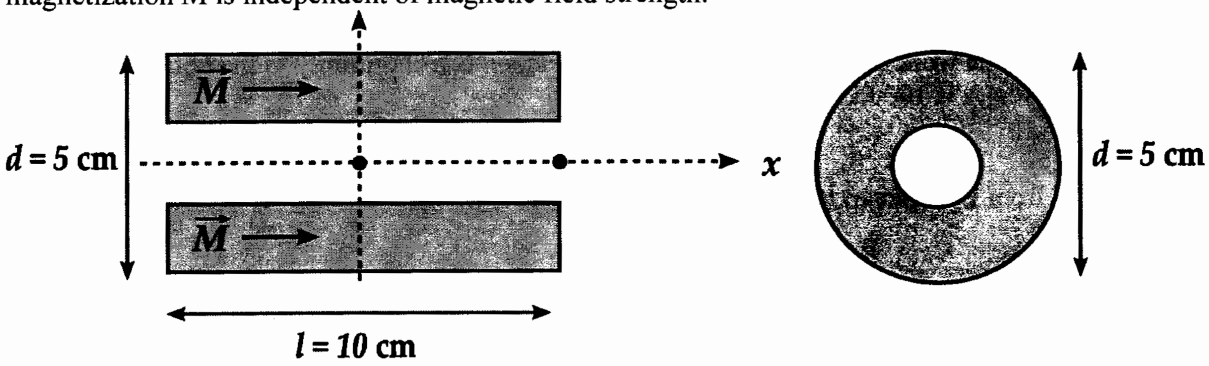

**3.** The above-mentioned result can be extended for a solenoid. Using the equation show that for the solenoid

$$B = \frac{\mu_0 NI}{2}(\cos\theta_1 - \cos\theta_2),$$

where N is the number of turns/m.



4. The remanence $B_r$ of uniformly magnetized permanent magnet is 1.2 T. The magnet has a cylindrical shape and is 5 cm in diameter (*d*) and 10 cm in length (*l*). The magnetization vector is parallel with x-axis. A hole of 1 cm in diameter is drilled in the center of the magnet (parallel with x-axis). Calculate the magnetic field B (also its direction) at x = 0 and x = 5 cm (see fig. below). Assume that the magnetization M is independent of magnetic field strength.




5. a) Define the voltage V induced between the wing tips of an aeroplane (wingspan is 60 m) when the velocity of the plane is perpendicular to magnetic field of the earth. The velocity of the aircraft is 1000 km/hour and you can assume a magnetic field of 50 μT. (4 p)

   b) Starting from equation $V = L\frac{dI}{dt}$, show that the energy of the magnetic field is

$$U = \frac{1}{2\mu_0} B^2 V.$$

   Note $L = \mu_0 N^2 \pi r^2 l$, where $l$ is the length of the solenoid, N turns/m and $V$ is the volume of the solenoid. (6 p)

6. The components of an elliptically polarized wave propagating in air are:
   $E_x = 3.0 \sin(\omega t - kz)$ $\mathrm{Vm}^{-1}$
   $E_y = 6.0 \sin(\omega t - kz + 5\pi/12)$ $\mathrm{Vm}^{-1}$.
   Calculate
   a) components of magnetic field (5 p)
   b) average power of the wave ($\mathrm{W/m}^2$). (5 p)

7. a) Show that at the boundary of two different media $H_{\parallel}$ is continuous (4 p)
   b) explain remanence (2 p)
   c) explain skin-effect (2 p)
   d) explain why TEM-mode cannot propagate inside the rectangular wave guide? Describe shortly the waves, which are capable of propagating inside the afore-mentioned waveguide. (2 p)

From: physics.nis

## Fundamental Physical Constants — Frequently used constants

| Quantity | Symbol | Value | Unit | Relative std. uncert. $u_r$ |
|---|---|---|---|---|
| speed of light in vacuum | $c, c_0$ | 299 792 458 | $\mathrm{m\,s^{-1}}$ | (exact) |
| magnetic constant | $\mu_0$ | $4\pi \times 10^{-7}$ | $\mathrm{N\,A^{-2}}$ | |
| | | $= 12.566\,370\,614... \times 10^{-7}$ | $\mathrm{N\,A^{-2}}$ | (exact) |
| electric constant $1/\mu_0 c^2$ | $\varepsilon_0$ | $8.854\,187\,817... \times 10^{-12}$ | $\mathrm{F\,m^{-1}}$ | (exact) |
| Newtonian constant of gravitation | $G$ | $6.673(10) \times 10^{-11}$ | $\mathrm{m^3\,kg^{-1}\,s^{-2}}$ | $1.5 \times 10^{-3}$ |
| Planck constant | $h$ | $6.626\,068\,76(52) \times 10^{-34}$ | J s | $7.8 \times 10^{-8}$ |
| $h/2\pi$ | $\hbar$ | $1.054\,571\,596(82) \times 10^{-34}$ | J s | $7.8 \times 10^{-8}$ |
| elementary charge | $e$ | $1.602\,176\,462(63) \times 10^{-19}$ | C | $3.9 \times 10^{-8}$ |
| magnetic flux quantum $h/2e$ | $\Phi_0$ | $2.067\,833\,636(81) \times 10^{-15}$ | Wb | $3.9 \times 10^{-8}$ |
| conductance quantum $2e^2/h$ | $G_0$ | $7.748\,091\,696(28) \times 10^{-5}$ | S | $3.7 \times 10^{-9}$ |
| electron mass | $m_e$ | $9.109\,381\,88(72) \times 10^{-31}$ | kg | $7.9 \times 10^{-8}$ |
| proton mass | $m_p$ | $1.672\,621\,58(13) \times 10^{-27}$ | kg | $7.9 \times 10^{-8}$ |
| proton-electron mass ratio | $m_p/m_e$ | $1\,836.152\,6675(39)$ | | $2.1 \times 10^{-9}$ |
| fine-structure constant $e^2/4\pi\varepsilon_0\hbar c$ | $\alpha$ | $7.297\,352\,533(27) \times 10^{-3}$ | | $3.7 \times 10^{-9}$ |
| inverse fine-structure constant | $\alpha^{-1}$ | $137.035\,999\,76(50)$ | | $3.7 \times 10^{-9}$ |
| Rydberg constant $\alpha^2 m_e c/2h$ | $R_\infty$ | $10\,973\,731.568\,549(83)$ | $\mathrm{m^{-1}}$ | $7.6 \times 10^{-12}$ |
| Avogadro constant | $N_A, L$ | $6.022\,141\,99(47) \times 10^{23}$ | $\mathrm{mol^{-1}}$ | $7.9 \times 10^{-8}$ |
| Faraday constant $N_A e$ | $F$ | $96\,485.3415(39)$ | $\mathrm{C\,mol^{-1}}$ | $4.0 \times 10^{-8}$ |
| molar gas constant | $R$ | $8.314\,472(15)$ | $\mathrm{J\,mol^{-1}\,K^{-1}}$ | $1.7 \times 10^{-6}$ |
| Boltzmann constant $R/N_A$ | $k$ | $1.380\,6503(24) \times 10^{-23}$ | $\mathrm{J\,K^{-1}}$ | $1.7 \times 10^{-6}$ |
| Stefan-Boltzmann constant $(\pi^2/60)k^4/\hbar^3c^2$ | $\sigma$ | $5.670\,400(40) \times 10^{-8}$ | $\mathrm{W\,m^{-2}\,K^{-4}}$ | $7.0 \times 10^{-6}$ |
| | | Non-SI units accepted for use with the SI | | |
| electron volt: $(e/\mathrm{C})$ J | eV | $1.602\,176\,462(63) \times 10^{-19}$ | J | $3.9 \times 10^{-8}$ |
| (unified) atomic mass unit $1\,\mathrm{u} = m_u = \frac{1}{12}m(^{12}\mathrm{C}) = 10^{-3}\,\mathrm{kg\,mol^{-1}}/N_A$ | u | $1.660\,538\,73(13) \times 10^{-27}$ | kg | $7.9 \times 10^{-8}$ |

## VECTOR IDENTITIES[4]

Notation: $f$, $g$, are scalars; $\mathbf{A}$, $\mathbf{B}$, etc., are vectors; $T$ is a tensor; $I$ is the unit dyad.

(1) $\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{A}\times\mathbf{B}\cdot\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = \mathbf{B}\times\mathbf{C}\cdot\mathbf{A} = \mathbf{C}\cdot\mathbf{A}\times\mathbf{B} = \mathbf{C}\times\mathbf{A}\cdot\mathbf{B}$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{C}\times\mathbf{B})\times\mathbf{A} = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C}$

(3) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) + \mathbf{B}\times(\mathbf{C}\times\mathbf{A}) + \mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = 0$

(4) $(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C})$

(5) $(\mathbf{A}\times\mathbf{B})\times(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\times\mathbf{B}\cdot\mathbf{D})\mathbf{C} - (\mathbf{A}\times\mathbf{B}\cdot\mathbf{C})\mathbf{D}$

(6) $\nabla(fg) = \nabla(gf) = f\nabla g + g\nabla f$

(7) $\nabla\cdot(f\mathbf{A}) = f\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla f$

(8) $\nabla\times(f\mathbf{A}) = f\nabla\times\mathbf{A} + \nabla f\times\mathbf{A}$

(9) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot\nabla\times\mathbf{A} - \mathbf{A}\cdot\nabla\times\mathbf{B}$

(10) $\nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A}) + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$

(11) $\mathbf{A}\times(\nabla\times\mathbf{B}) = (\nabla\mathbf{B})\cdot\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$

(12) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(13) $\nabla^2 f = \nabla\cdot\nabla f$

(14) $\nabla^2\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla\times\nabla\times\mathbf{A}$

(15) $\nabla\times\nabla f = 0$

(16) $\nabla\cdot\nabla\times\mathbf{A} = 0$

If $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are orthonormal unit vectors, a second-order tensor $T$ can be written in the dyadic form

(17) $T = \sum_{i,j} T_{ij}\mathbf{e}_i\mathbf{e}_j$

In cartesian coordinates the divergence of a tensor is a vector with components

(18) $(\nabla\cdot T)_i = \sum_j(\partial T_{ji}/\partial x_j)$

[This definition is required for consistency with Eq. (29)]. In general

(19) $\nabla\cdot(\mathbf{A}\mathbf{B}) = (\nabla\cdot\mathbf{A})\mathbf{B} + (\mathbf{A}\cdot\nabla)\mathbf{B}$

(20) $\nabla\cdot(fT) = \nabla f\cdot T + f\nabla\cdot T$