FYSA220 Sähköoppi   Tentti 30. 8. 2013

1. Varausjakauma $\rho(\bar{r})$ on vakio R-säteisessä pallossa. Laske $\overline{E}(\bar{r})$ ja $\phi(\bar{r})$, kun r < R ja r > R.

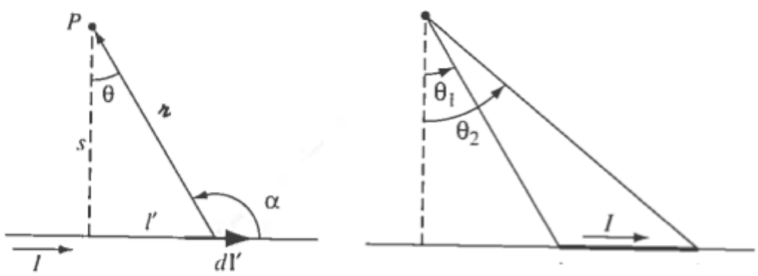
2. Määritä kahden samankeskisen onton johdepallon kapasitanssi C. Sisemmän pallon säde on a ja ulomman b.

3. Positiivinen varaus q on etäisyydellä d tasaisesta ja laajasta metallilevystä. Määritä metallin pintaan indusoitunut pintavaraustiheys σ.

4. Osoita Biot-Savartin lauseen avulla, että kuvan 1 asetelman mukaisessa tilanteessa suoran virtajohtimen (kulmat määrittävät johtimen pituuden), jossa kulkee virta I, tuottama magneettikenttä pisteessä P on

$$B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1).$$

Vihje: Muista, että $tan\theta$ derivaatta on $1/cos^2\theta$.



Kuva 1

5. Laserin keskimääräinen valoteho on 100 mW. Jos intensiteetti on tasaisesti jakautunut ja pyöreän valokeilan halkaisija on 0.5 mm kuinka suuret ovat ***E***- ja ***B***-kenttien amplitudit?

Fundamental Physical Constants — Frequently used constants

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
speed of light in vacuum	c, c_0	299 792 458	m s^{-1}	(exact)
magnetic constant	μ_0	$4\pi \times 10^{-7}$	N A^{-2}	
		$= 12.566\,370\,614... \times 10^{-7}$	N A^{-2}	(exact)
electric constant $1/\mu_0 c^2$	ϵ_0	$8.854\,187\,817... \times 10^{-12}$	F m^{-1}	(exact)
Newtonian constant of gravitation	G	$6.673(10) \times 10^{-11}$	m^3 kg^{-1} s^{-2}	1.5×10^{-3}
Planck constant	h	$6.626\,068\,76(52) \times 10^{-34}$	J s	7.8×10^{-8}
$h/2\pi$	$\hbar$	$1.054\,571\,596(82) \times 10^{-34}$	J s	7.8×10^{-8}
elementary charge	e	$1.602\,176\,462(63) \times 10^{-19}$	C	3.9×10^{-8}
magnetic flux quantum $h/2e$	Φ_0	$2.067\,833\,636(81) \times 10^{-15}$	Wb	3.9×10^{-8}
conductance quantum $2e^2/h$	G_0	$7.748\,091\,696(28) \times 10^{-5}$	S	3.7×10^{-9}
electron mass	m_e	$9.109\,381\,88(72) \times 10^{-31}$	kg	7.9×10^{-8}
proton mass	m_p	$1.672\,621\,58(13) \times 10^{-27}$	kg	7.9×10^{-8}
proton-electron mass ratio	m_p/m_e	1 836.152 6675(39)		2.1×10^{-9}
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,533(27) \times 10^{-3}$		3.7×10^{-9}
inverse fine-structure constant	α^{-1}	137.035 999 76(50)		3.7×10^{-9}
Rydberg constant $\alpha^2 m_e c/2h$	R_∞	10 973 731.568 549(83)	m^{-1}	7.6×10^{-12}
Avogadro constant	N_A, L	$6.022\,141\,99(47) \times 10^{23}$	mol^{-1}	7.9×10^{-8}
Faraday constant $N_A e$	F	96 485.3415(39)	C mol^{-1}	4.0×10^{-8}
molar gas constant	R	8.314 472(15)	J mol^{-1} K^{-1}	1.7×10^{-6}
Boltzmann constant R/N_A	k	$1.380\,6503(24) \times 10^{-23}$	J K^{-1}	1.7×10^{-6}
Stefan-Boltzmann constant $(\pi^2/60)k^4/\hbar^3c^2$	σ	$5.670\,400(40) \times 10^{-8}$	W m^{-2} K^{-4}	7.0×10^{-6}
	Non-SI units accepted for use with the SI			
electron volt: (e/C) J	eV	$1.602\,176\,462(63) \times 10^{-19}$	J	3.9×10^{-8}
(unified) atomic mass unit $1\ \mathrm{u} = m_u = \frac{1}{12}m(^{12}\mathrm{C}) = 10^{-3}$ kg mol$^{-1}/N_A$	u	$1.660\,538\,73(13) \times 10^{-27}$	kg	7.9×10^{-8}

VECTOR IDENTITIES[4]

Notation: f, g, are scalars; **A**, **B**, etc., are vectors; T is a tensor; I is the unit dyad.

(1) $\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{A}\times\mathbf{B}\cdot\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = \mathbf{B}\times\mathbf{C}\cdot\mathbf{A} = \mathbf{C}\cdot\mathbf{A}\times\mathbf{B} = \mathbf{C}\times\mathbf{A}\cdot\mathbf{B}$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{C}\times\mathbf{B})\times\mathbf{A} = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C}$

(3) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) + \mathbf{B}\times(\mathbf{C}\times\mathbf{A}) + \mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = 0$

(4) $(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C})$

(5) $(\mathbf{A}\times\mathbf{B})\times(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\times\mathbf{B}\cdot\mathbf{D})\mathbf{C} - (\mathbf{A}\times\mathbf{B}\cdot\mathbf{C})\mathbf{D}$

(6) $\nabla(fg) = \nabla(gf) = f\nabla g + g\nabla f$

(7) $\nabla\cdot(f\mathbf{A}) = f\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla f$

(8) $\nabla\times(f\mathbf{A}) = f\nabla\times\mathbf{A} + \nabla f\times\mathbf{A}$

(9) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot\nabla\times\mathbf{A} - \mathbf{A}\cdot\nabla\times\mathbf{B}$

(10) $\nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A}) + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$

(11) $\mathbf{A}\times(\nabla\times\mathbf{B}) = (\nabla\mathbf{B})\cdot\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$

(12) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(13) $\nabla^2 f = \nabla\cdot\nabla f$

(14) $\nabla^2\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla\times\nabla\times\mathbf{A}$

(15) $\nabla\times\nabla f = 0$

(16) $\nabla\cdot\nabla\times\mathbf{A} = 0$

If $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are orthonormal unit vectors, a second-order tensor T can be written in the dyadic form

(17) $T = \sum_{i,j} T_{ij}\mathbf{e}_i\mathbf{e}_j$

In cartesian coordinates the divergence of a tensor is a vector with components

(18) $(\nabla\cdot T)_i = \sum_j(\partial T_{ji}/\partial x_j)$

[This definition is required for consistency with Eq. (29)]. In general

(19) $\nabla\cdot(\mathbf{A}\mathbf{B}) = (\nabla\cdot\mathbf{A})\mathbf{B} + (\mathbf{A}\cdot\nabla)\mathbf{B}$

(20) $\nabla\cdot(fT) = \nabla f\cdot T + f\nabla\cdot T$