Exam: 4 problems, 4 hours

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1. (a) (3p.) An infinitesimal rotation by angle ϵ around the z-axis is represented by the operator

$$R_z(\epsilon) = 1 - \frac{i\epsilon}{\hbar} J_z.$$

Starting from this, derive the operator representing a finite rotation by angle θ around the z-axis. You can assume known that

$$\lim_{N\to\infty} \left(1 + \frac{x}{N}\right)^N = e^x.$$

(b) (3p.) Consider the operator $T(a) = e^{-iPa/\hbar}$. Using series expansion, show that operating on a wavefunction $\psi(x)$ with $T(a)$ gives

$$T(a)\psi(x) = \psi(x - a).$$

Why does this result show that $T(a)$ represents a translation by a to right (i.e. to the direction of positive x-axis)?

2. Consider angular momentum operator $\vec{J}$, whose components satisfy the commutation relations

$$[J_x, J_y] = i\hbar J_z, \qquad [J_y, J_z] = i\hbar J_x, \qquad [J_z, J_x] = i\hbar J_y.$$

(a) (2p.) The square of the angular momentum is

$$J^2 = J_x^2 + J_y^2 + J_z^2.$$

Show that each component of $\vec{J}$ commutes with J^2.

(b) (2p.) Define operators

$$J_\pm = J_x \pm iJ_y.$$

Show, that

$$[J_z, J_\pm] = \pm\hbar J_\pm, \qquad [J^2, J_\pm] = 0$$

and

$$J_+J_- = J^2 - J_z^2 - \hbar J_z, \qquad J_-J_+ = J^2 - J_z^2 + \hbar J_z.$$

(c) (2p.) On the basis of the result obtained in part (a) we know that J^2 and one component of $\vec{J}$ – let this component be J_z – have common eigenstates. Denote these eigenstates by $|jm\rangle$ and assume that they satisfy

$$J^2|jm\rangle = j(j+1)\hbar^2|jm\rangle, \qquad J_z|jm\rangle = m\hbar|jm\rangle.$$

Show that J_+ raises and J_- lowers the eigenvalue of J_z by the amount $\hbar$. Show also that J_+ and J_- do not change the eigenvalue of J^2. In other words, show that

$$J_zJ_\pm|jm\rangle = (m \pm 1)\hbar J_\pm|jm\rangle, \qquad J^2J_\pm|jm\rangle = j(j+1)\hbar^2 J_\pm|jm\rangle.$$

3. Consider a system consisting of a spin-1 particle (particle 1) and a spin-1/2 particle (particle 2).

   (a) (2p.) Write all the states in the coupled basis in terms of the uncoupled states. (You may look up the Clebsch–Gordan coefficients from the table.)

   (b) (2p.) Let the Hamiltonian of the system be

   $$H = \frac{E_0}{\hbar^2}\vec{S}_1 \cdot \vec{S}_2,$$

   where E_0 is a constant, and $\vec{S}_1$ and $\vec{S}_2$ are the spin operators of particles 1 and 2. If the energy of the system is measured, what results can be obtained?

   (c) (2p.) Suppose that the spins of both particles have been measured along the z-axis, and the results $+\hbar$ for particle 1 and $+\hbar/2$ for particle 2 have been found. If the energy of the system is measured immediately after this, what are the probabilities for obtaining each possible value of the energy?

4. (a) (3p.) Consider a harmonic oscillator placed on a weak electric field. Then the unperturbed Hamiltonian is $H_0 = P^2/2m + \frac{1}{2}m\omega^2 x^2$ and the perturbation is $V = -q|\vec{E}|x$. Using perturbation theory, calculate the first- and second-order corrections to the energy of the ground state $|0\rangle$.

   (b) (3p.) Use the variational method to estimate the ground state energy of the one-dimensional potential $V(x) = \lambda x^4$. Choose the function $\psi_\alpha(x) = e^{-\alpha x^2}$ as your trial function.

Hyötytietoa:

$$e^{ix} = \cos x + i \sin x \qquad e^{i\pi} + 1 = 0$$

$$\sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right) \qquad \cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$$

$$\sin(2x) = 2 \sin x \cos x \qquad \cos(2x) = \cos^2 x - \sin^2 x$$

$$\frac{1}{2}(1 - \cos x) = \sin^2 \frac{x}{2} \qquad \frac{1}{2}(1 + \cos x) = \cos^2 \frac{x}{2}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n f(x_0)}{dx^n}(x - x_0)^n \qquad e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\int_0^{\infty} dx\, x^n e^{-cx} = \frac{n!}{c^{n+1}} \qquad \int_{-\infty}^{\infty} dx\, e^{-cx^2} = \sqrt{\frac{\pi}{c}}$$

$$\int_{-\infty}^{\infty} dx\, x^2 e^{-cx^2} = \frac{1}{2}\sqrt{\frac{\pi}{c^3}} \qquad \int_{-\infty}^{\infty} dx\, x^4 e^{-cx^2} = \frac{3}{4}\sqrt{\frac{\pi}{c^5}}$$

$$\int_{-\infty}^{\infty} dx\, \delta(x - x_0) f(x) = f(x_0) \qquad \int_{-\infty}^{\infty} dx\, \delta(x - x_0) = 1$$

$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx\, e^{-ikx} f(x) \qquad f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx\, e^{ikx} \tilde{f}(k)$$

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

$$U(t) = e^{-iHt/\hbar}$$

$$\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2} \qquad \Delta A \Delta B \geq \frac{1}{2}|\langle [A, B] \rangle|$$

$$\Delta x \Delta p \geq \frac{\hbar}{2} \qquad \Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\frac{d\langle A \rangle}{dt} = -\frac{i}{\hbar}\langle [A, H] \rangle + \left\langle \frac{\partial A}{\partial t} \right\rangle$$

$$H = \frac{P^2}{2m} + V(x) \qquad [X, P] = i\hbar$$

$$\psi(x) = \langle x|\psi\rangle \qquad X = x \qquad P = -i\hbar \frac{d}{dx}$$

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$$

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

$$-\frac{\hbar^2}{2m}\frac{d^2 \psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$c = 2.998 \cdot 10^8 \text{ m/s} \qquad \hbar = 1.055 \cdot 10^{-34} \text{ Js} \qquad q_e = 1.602 \cdot 10^{-19} \text{ C}$$

$$\epsilon_0 = 8.854 \cdot 10^{-12} \frac{\text{C}^2}{\text{Nm}^2} \qquad \mu_0 = 4\pi \cdot 10^{-7} \frac{\text{N}}{\text{A}^2} \qquad m_e = 9.109 \cdot 10^{-31} \text{ kg}$$

$$U(t) = e^{-iHt/\hbar} \qquad T(\vec{a}) = e^{-i\vec{P}\cdot\vec{a}/\hbar} \qquad R_{\vec{n}}(\theta) = e^{-i\theta\vec{n}\cdot\vec{J}/\hbar}$$

$$[J_x, J_y] = i\hbar J_z \qquad [J_y, J_z] = i\hbar J_x \qquad [J_z, J_x] = i\hbar J_y$$

$$J^2 = J_x^2 + J_y^2 + J_z^2 \qquad J_\pm = J_x \pm iJ_y$$

$$[J_z, J^2] = 0 \qquad [J_z, J_\pm] = \pm\hbar J_\pm \qquad [J_+, J_-] = 2\hbar J_z$$

$$J^2|jm\rangle = \hbar^2 j(j+1)|jm\rangle \qquad J_z|jm\rangle = m\hbar|jm\rangle$$

$$J_\pm|jm\rangle = \hbar\sqrt{j(j+1) - m(m\pm 1)}|j, m\pm 1\rangle$$

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma} \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_i\sigma_j - \sigma_j\sigma_i = 2i\sum_k \epsilon_{ijk}\sigma_k \qquad \sigma_i\sigma_j + \sigma_j\sigma_i = 2\delta_{ij}$$

$$H = -\vec{\mu}\cdot\vec{B} \qquad \vec{\mu} = \gamma\vec{S} = g\frac{q}{2m}\vec{S} \qquad g_e \simeq 2$$

$$|j_1 j_2, jm\rangle = \sum_{m_1, m_2} \langle j_1 m_1, j_2 m_2|jm\rangle |j_1 m_1\rangle |j_2 m_2\rangle$$

$$E_n^{(1)} = \langle\psi_n^{(0)}|V|\psi_n^{(0)}\rangle$$

$$E_n^{(2)} = \sum_{m\neq q} \frac{|\langle\psi_m^{(0)}|V|\psi_n^{(0)}\rangle|^2}{E_n^{(0)} - E_m^{(0)}} \qquad |\psi_n^{(1)}\rangle = \sum_{m\neq q} \frac{\langle\psi_m^{(0)}|V|\psi_n^{(0)}\rangle^2}{E_n^{(0)} - E_m^{(0)}} |\psi_m^{(0)}\rangle$$

Spin-1 –hiukkasen spinoperaattorin komponentit:

$$S_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad S_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad S_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Harmoninen oskillaattori:

$$H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 X^2 = \hbar\omega(a^\dagger a + \tfrac{1}{2})$$

$$a = \sqrt{\frac{m\omega}{2\hbar}}X + \frac{i}{\sqrt{2\hbar m\omega}}P \qquad a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}X - \frac{i}{\sqrt{2\hbar m\omega}}P$$

$$a|n\rangle = \sqrt{n}|n-1\rangle \qquad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle \qquad H|n\rangle = \hbar\omega(n+\tfrac{1}{2})|n\rangle$$

Vetyatomi:

$$a_0 = \frac{\hbar^2}{me^2} = 5.29\times 10^{-11}\ \text{m} \qquad E_0 = \frac{me^4}{2\hbar^2} = 13.6\ \text{eV}$$

$$\begin{aligned}
\psi_{100}(r) &= \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \\
\psi_{200}(r) &= \frac{1}{\sqrt{32\pi a_0^3}}\left(2 - \frac{r}{a_0}\right) e^{-r/2a_0} \\
\psi_{210}(r,\theta) &= \frac{1}{\sqrt{32\pi a_0^3}}\frac{r}{a_0} e^{-r/2a_0}\cos\theta \\
\psi_{21\pm 1}(r,\theta,\phi) &= \mp\frac{1}{\sqrt{32\pi a_0^3}}\frac{r}{a_0} e^{-r/2a_0}\sin\theta e^{\pm i\phi}
\end{aligned}$$

34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2			
m_1	m_2	Coefficients		
.	.			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

1/2×1/2

m_1	m_2	1	0
		+1	
+1/2	+1/2	1	
		1	0
		0	0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2
		1	
		−1	
−1/2	−1/2	1	

1×1/2

m_1	m_2	J	J
		3/2	
		+3/2	
+1	+1/2	1	
		3/2	1/2
		+1/2	+1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3
		3/2	1/2
		−1/2	−1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3
		3/2	
		−3/2	
−1	−1/2	1	

2×1/2

m_1	m_2	J	J
		5/2	
		+5/2	
+2	+1/2	1	
		5/2	3/2
		+3/2	+3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5
		5/2	3/2
		+1/2	+1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5
		5/2	3/2
		−1/2	−1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5
		5/2	3/2
		−3/2	−3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5
		5/2	
		−5/2	
−2	−1/2	1	

3/2×1/2

m_1	m_2	J	J
		2	
		+2	
+3/2	+1/2	1	
		2	1
		+1	+1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4
		2	1
		0	0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2
		2	1
		−1	−1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4
		2	
		−2	
−3/2	−1/2	1	

2×1

m_1	m_2	J	J	J
		3		
		+3		
+2	+1	1		
		3	2	
		+2	+2	
+2	0	1/3	2/3	
+1	+1	2/3	−1/3	
		3	2	1
		+1	+1	+1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10
		3	2	1
		0	0	0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10
		3	2	1
		−1	−1	−1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5
		3	2	
		−2	−2	
−1	−1	2/3	1/3	
−2	0	1/3	−2/3	
		3		
		−3		
−2	−1	1		

1×1

m_1	m_2	J	J	J
		2		
		+2		
+1	+1	1		
		2	1	
		+1	+1	
+1	0	1/2	1/2	
0	+1	1/2	−1/2	
		2	1	0
		0	0	0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3
		2	1	
		−1	−1	
0	−1	1/2	1/2	
−1	0	1/2	−1/2	
		2		
		−2		
−1	−1	1		

3/2×1

m_1	m_2	J	J	J
		5/2		
		+5/2		
+3/2	+1	1		
		5/2	3/2	
		+3/2	+3/2	
+3/2	0	2/5	3/5	
+1/2	+1	3/5	−2/5	
		5/2	3/2	1/2
		+1/2	+1/2	+1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6
		5/2	3/2	1/2
		−1/2	−1/2	−1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2
		5/2	3/2	
		−3/2	−3/2	
−1/2	−1	3/5	2/5	
−3/2	0	2/5	−3/5	
		5/2		
		−5/2		
−3/2	−1	1		

3/2×3/2

m_1	m_2	J	J	J	J
		3			
		+3			
+3/2	+3/2	1			
		3	2		
		+2	+2		
+3/2	+1/2	1/2	1/2		
+1/2	+3/2	1/2	−1/2		
		3	2	1	
		+1	+1	+1	
+3/2	−1/2	1/5	1/2	3/10	
+1/2	+1/2	3/5	0	−2/5	
−1/2	+3/2	1/5	−1/2	3/10	
		3	2	1	0
		0	0	0	0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4
		3	2	1	
		−1	−1	−1	
+1/2	−3/2	1/5	1/2	3/10	
−1/2	−1/2	3/5	0	−2/5	
−3/2	+1/2	1/5	−1/2	3/10	
		3	2		
		−2	−2		
−1/2	−3/2	1/2	1/2		
−3/2	−1/2	1/2	−1/2		
		3			
		−3			
−3/2	−3/2	1			

2×3/2

m_1	m_2	J	J	J	J
		7/2			
		+7/2			
+2	+3/2	1			
		7/2	5/2		
		+5/2	+5/2		
+2	+1/2	3/7	4/7		
+1	+3/2	4/7	−3/7		
		7/2	5/2	3/2	
		+3/2	+3/2	+3/2	
+2	−1/2	1/7	16/35	2/5	
+1	+1/2	4/7	1/35	−2/5	
0	+3/2	2/7	−18/35	1/5	
		7/2	5/2	3/2	1/2
		+1/2	+1/2	+1/2	+1/2
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10
		7/2	5/2	3/2	1/2
		−1/2	−1/2	−1/2	−1/2
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5
		7/2	5/2	3/2	
		−3/2	−3/2	−3/2	
0	−3/2	2/7	18/35	1/5	
−1	−1/2	4/7	−1/35	−2/5	
−2	+1/2	1/7	−16/35	2/5	
		7/2	5/2		
		−5/2	−5/2		
−1	−3/2	4/7	3/7		
−2	−1/2	3/7	−4/7		
		7/2			
		−7/2			
−2	−3/2	1			

2×2

m_1	m_2	J	J	J	J	J
		4				
		+4				
+2	+2	1				
		4	3			
		+3	+3			
+2	+1	1/2	1/2			
+1	+2	1/2	−1/2			
		4	3	2		
		+2	+2	+2		
+2	0	3/14	1/2	2/7		
+1	+1	4/7	0	−3/7		
0	+2	3/14	−1/2	2/7		
		4	3	2	1	
		+1	+1	+1	+1	
+2	−1	1/14	3/10	3/7	1/5	
+1	0	3/7	1/5	−1/14	−3/10	
0	+1	3/7	−1/5	−1/14	3/10	
−1	+2	1/14	−3/10	3/7	−1/5	
		4	3	2	1	0
		0	0	0	0	0
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5
		4	3	2	1	
		−1	−1	−1	−1	
+1	−2	1/14	3/10	3/7	1/5	
0	−1	3/7	1/5	−1/14	−3/10	
−1	0	3/7	−1/5	−1/14	3/10	
−2	+1	1/14	−3/10	3/7	−1/5	
		4	3	2		
		−2	−2	−2		
0	−2	3/14	1/2	2/7		
−1	−1	4/7	0	−3/7		
−2	0	3/14	−1/2	2/7		
		4	3			
		−3	−3			
−1	−2	1/2	1/2			
−2	−1	1/2	−1/2			
		4				
		−4				
−2	−2	1				

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^1_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,2} = \Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2} \qquad d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta \qquad d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2} \qquad d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2} \qquad d^2_{2,-2} = \Big(\frac{1-\cos\theta}{2}\Big)^2 \qquad d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1) \qquad d^2_{0,0} = \Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

Figure 34.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.