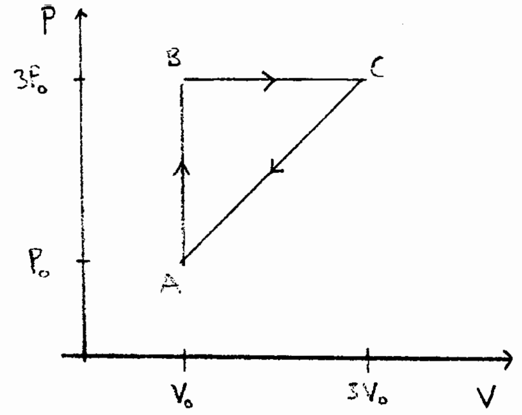
FYSA241 STATISTICAL PHYSICS PART 1
Examination on 17 September 2010

1. One mole of ideal gas undergoes quasistatically the closed cycle indicated in the figure below.



   Find the work done and the heat absorbed by the gas during this cycle in terms of the given P and V variables.

2. Calculate the change in the internal energy of the system, when 1 mole of water is evaporated at 1 atm pressure and 100°C temperature into water vapour at the same temperature and pressure. Under these conditions the molar volume of water is $1.88 \cdot 10^{-5}$ m^3/mol, that of water vapour is $3.02 \cdot 10^{-2}$ m^3/mol, and the latent heat of evaporation of water is $4.06 \cdot 10^4$ J/mol.

3. N independent magnetic dipoles (spin-$\frac{1}{2}$ dipoles) of equal dipole moment μ are in an external magnetic field $\mathbf{B}$. The statistical weight of a state of the system in which n dipoles point in the direction opposite to the magnetic field is given by
$$\Omega(n) = \frac{N!}{n!\,(N-n)!}.$$
   The energy of the system is then $E(n) = -n\mu B + (N-n)\mu B$. If this state is an equilibrium state of the system, what is then its temperature?

4. A mole of thermally isolated ideal gas, whose temperature is 300 K, expands quasistatically into four times of its initial volume. What is the related change in the entropy of the system?

5. A monoatomis gas that obeys the van der Waals equation of state has a molar internal energy of $E = \frac{3}{2}RT - a/V$, in which V is the molar volume a is a constant. Initially the gas is in the state (T_1, V_1) and expands then adiabatically into vacuum such that its volume becomes V_2. What is the temperature of the gas (T_2) at the end of this process?

6. Explain the operation principle of the refrigerator and show that its best possible efficiency is $\eta \equiv Q_2/W = T_2/(T_1 - T_2)$, in which Q_2 is the amount of heat removed from the refrigerator at temperature T_2 into the environment at temperature $T_1 > T_2$, and W is the amount of work used for that process.

Mahdollisesti hyödyllisiä tietoja

$$k_B = 1.3805 \times 10^{-23}\ \mathrm{JK^{-1}} \qquad k_B \cdot 300\ \mathrm{K} \approx \frac{1}{40}\ \mathrm{eV} \qquad R = 8.3143\ \mathrm{J/mol\,K}$$

$$0\ ^\circ\mathrm{C} = 273.15\ \mathrm{K} \qquad c = 3 \times 10^8\ \mathrm{m/s} \qquad m_e c^2 = 511\ \mathrm{keV}$$

$$1\ \mathrm{atm} = 1.013 \times 10^5\ \mathrm{Nm^{-2}} \qquad g = 9.81\ \mathrm{ms^{-2}} \qquad \hbar = 1.0545 \times 10^{-34}\ \mathrm{Js}$$

$$\mathrm{d}E = đQ + đW \qquad \mathrm{d}E = T\mathrm{d}S - P\mathrm{d}V \qquad \mathrm{d}E = T\mathrm{d}S - P\mathrm{d}V + \mu\mathrm{d}N$$

$$S = k_B \ln\Omega \qquad \ln n! \sim n\ln n - n \qquad \binom{n}{m} = \frac{n!}{m!(n-m)!}$$

$$S = -k_B \sum_r p_r \ln p_r \qquad p_r = \frac{1}{Z} e^{-\beta E_r} \qquad Z = \sum_r e^{-\beta E_r} \qquad \beta = 1/k_B T$$

$$G = E - TS + PV \qquad F = E - TS \qquad F = -k_B T \ln Z \qquad E = -\frac{\partial}{\partial\beta}\ln Z$$

$$PV = Nk_B T = nRT \qquad \left(\frac{\mathrm{d}P}{\mathrm{d}T}\right)_{\mathrm{cx}} = \frac{\Delta S}{\Delta V} = \frac{L_{12}}{T\Delta V}$$

$$Z_{\mathrm{cl}} = \frac{1}{N!} Z_1^N \qquad Z_1^{\mathrm{tr}} = V\left(\frac{mk_B T}{2\pi\hbar^2}\right)^{3/2} \qquad P(v)\mathrm{d}v = \sqrt{\frac{2}{\pi}}\left(\frac{m}{k_B T}\right)^{3/2} v^2 \exp\left[\frac{-mv^2}{2k_B T}\right]\mathrm{d}v$$

$$p_{N,r} = \frac{1}{\mathcal{Z}} e^{\beta(\mu N - E_{N,r})} \qquad \mathcal{Z} = \sum_{N,r} e^{\beta(\mu N - E_{N,r})} \qquad \Omega = -k_B T \ln \mathcal{Z} = E - TS - \mu N = -PV$$

$$\mathcal{Z} = \prod_r \mathcal{Z}_r = \prod_r \sum_{n_r=0}^{\infty} e^{\beta(\mu - \varepsilon_r)n_r} \qquad \langle n_r \rangle = \frac{1}{e^{\beta(\varepsilon_r - \mu)} \pm 1}$$

$$u(\omega, T)\mathrm{d}\omega = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3 \mathrm{d}\omega}{e^{\beta\hbar\omega} - 1} \qquad \mu = \varepsilon_F\left[1 - \frac{\pi^2}{12}\left(\frac{T}{T_F}\right)^2 + \ldots\right] \qquad k_B T_C \approx 3.3 \times \frac{\hbar^2}{m}\left(\frac{N}{V}\right)^{2/3}$$

$$f(x,y,z) = 0 \quad \text{"}\Rightarrow\text{"} \quad \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1 \quad \text{ja} \quad \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial x}\right)_z = 1$$

$$\sinh x = \frac{1}{2}(e^x - e^{-x}) \qquad \cosh x = \frac{1}{2}(e^x + e^{-x}) \qquad \tanh x = \frac{\sinh x}{\cosh x}$$

$$1 + x + x^2 + \ldots = \frac{1}{1-x} \quad (|x| < 1) \qquad e^x = 1 + x + \frac{1}{2!}x^2 + \ldots \qquad (1+x)^q = 1 + qx + \frac{1}{2!}q(q-1)x^2 + \ldots$$