FYSH300 Particle physics 9.11.2012

1. half-course exam (1. välikoe): 4 problems, **4 hours**. Return the CG table and the particle tables together with your answer sheets!

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1. a) What is the experimental definition of a cross section? (1 point)

b) The decay $p \to e^+ + \mu^- + D^0$ has not been observed. Which four conservation laws would such a reaction violate, assuming a free, physical, proton? (1p)

c) Draw one possible Feynman graph of the following processes, to lowest order in the couplings. In your graph, mark clearly the particles, and the momentum & fermion(particle) arrows.

i) $e^+ + e^- \to \mu^+ + \mu^- + \mu^+ + \mu^-$ where both an electromagnetic and a weak interaction take place. (1p)

ii) $D^+ \to K^- + \pi^+ + \mu^+ + \nu_\mu$ (1p)

d) Consider a strong interaction process $K^- + p \to \Sigma^+ + \pi^-$.

i) Draw a quark diagram for this, and show that a resonance formation is possible. What is the quark content of the resonance? (1p)

ii) Suppose that you should experimentally determine the mass M_R and the mean lifetime τ of the resonance in this reaction. Sketch a figure of the cross section vs. the CMS energy in this process, and explain how M_R and τ can be determined from such a measurement (= from your figure). For drawing the figure, you may assume that the background to the resonance formation process is negligible. (1p)

2. a) The deuteron, which is a bound state of a proton p and a neutron n, is known to be of spin 1 and positive parity, i.e. a $J^P = 1^+$ state. Given this information, and recalling that p and n are $J^P = \frac{1}{2}^+$ particles, show that the deuteron can exist only in the $^{2S+1}L_J = {}^3\mathrm{S}_1$ and $^3\mathrm{D}_1$ states of the pn system. [Recall the notation: S is $L = 0$ and D is $L = 2$.] (3p)

b) Consider the electromagnetic decay $\eta \to \gamma + \gamma$ which is observed, and the decays $\eta \to \gamma + \gamma + \gamma$ and $\eta \to \pi^0\gamma$ which have not been observed.

i) Show that the states $|\gamma\gamma\rangle$, $|\gamma\gamma\gamma\rangle$ and $|\pi^0\gamma\rangle$ are eigenstates of C-parity. (1p)

ii) Using the above experimental information, determine the C-parities of η, γ and π^0. (2p)

3. Let's consider here an elastic process $e + p \to e + p$ in the colliding beams experiments at the DESY-HERA accelerator, where the energy of the electron was $E_1 = 30$ GeV and the energy of the proton $E_2 = 920$ GeV. In this ultrarelativistic case $m_e, m_p \ll E_1, E_2$, so that you can set the particle masses here to zero.

a) What is the CMS energy of these collisions? (1p)

Suppose then that in the CMS frame we observe that the electron scattered into an angle $\theta_3^* = 60°$, i.e. that $\cos\theta_3^* = +\frac{1}{2}$.

b) Sketch a figure of the momentum vectors and scattering angles in this scattering

i) in the CMS frame, and

ii) in the collider frame where the energies of the beams are the E_1 and E_2 mentioned above.

(1p)

c) Starting from the definitions of the Mandelstam variables s, t and u, compute the scattering angle θ_3 and the energy E_3 of the scattered electron in the collider frame where the energies of the beams are the E_1 and E_2 mentioned above. Express $\cos\theta_3$ and E_3 in terms of the energies E_1 and E_2 and compute also the numerical values of θ_3 and E_3. (4p)

[Recall that with our choice of metrics, the scalar product of two 4-vectors is $A^\mu B_\mu = A^0 B^0 - \mathbf{A} \cdot \mathbf{B}$]

4. According to the PDG particle tables, the full decay width of the $\Delta(1600)$ resonance is 350 MeV, the mass 1600 MeV, and the branching ratio to the $\Delta\pi$ channels altogether 50%.

a) How does the above information indicate to you that we can use the isospin symmetry to consider the physics of this decay prosess? (0.5 p)

b) We recall that the isospin states of the Δ quadret of $I = \frac{3}{2}$ are

$$\begin{aligned} |\Delta^{++}\rangle &= |I = \tfrac{3}{2}\,, I_3 = +\tfrac{3}{2}\,\rangle \\ |\Delta^{+}\rangle &= |I = \tfrac{3}{2}\,, I_3 = +\tfrac{1}{2}\,\rangle \\ |\Delta^{0}\rangle &= |I = \tfrac{3}{2}\,, I_3 = -\tfrac{1}{2}\,\rangle \\ |\Delta^{-}\rangle &= |I = \tfrac{3}{2}\,, I_3 = -\tfrac{3}{2}\,\rangle \end{aligned}$$

and the corresponding pion triplet states are

$$\begin{aligned} |\pi^{+}\rangle &= -|I = 1, I_3 = +1\rangle \\ |\pi^{0}\rangle &= |I = 1, I_3 = 0\rangle \\ |\pi^{-}\rangle &= |I = 1, I_3 = -1\rangle. \end{aligned}$$

Using the isospin symmetry, compute the decay width ratios

$$\frac{\Gamma(\Delta^+(1600) \to \Delta^+(1232) + \pi^0)}{\Gamma(\Delta^+(1600) \to \Delta^0(1232) + \pi^+)} \quad \text{and} \quad \frac{\Gamma(\Delta^+(1600) \to \Delta^{++}(1232) + \pi^-)}{\Gamma(\Delta^+(1600) \to \Delta^0(1232) + \pi^+)}$$

(4p)
[Recall that $\Gamma(i \to f) = K \int d\Omega |\langle f|\hat{T}|i\rangle|^2$ where K is a kinematical factor of masses and external momenta.]

c) Using the given information and the result you obtained above, calculate the decay width $\Gamma(\Delta^+(1600) \to \Delta^0(1232) + \pi^+)$, and give the result in MeVs. (1.5p)

35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND *d* FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

		J	J	…
		M	M	…
m_1	m_2	Coefficients		
m_1	m_2			
⋮	⋮			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

1/2×1/2

m_1	m_2	1
		+1
+1/2	+1/2	1

m_1	m_2	1	0
		0	0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	1
		−1
−1/2	−1/2	1

1×1/2

m_1	m_2	3/2
		+3/2
+1	+1/2	1

m_1	m_2	3/2	1/2
		+1/2	+1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	3/2	1/2
		−1/2	−1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	3/2
		−3/2
−1	−1/2	1

2×1

m_1	m_2	3
		+3
+2	+1	1

m_1	m_2	3	2
		+2	+2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	3	2	1
		+1	+1	+1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

m_1	m_2	3	2	1
		0	0	0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	3	2	1
		−1	−1	−1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	3	2
		−2	−2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	3
		−3
−2	−1	1

1×1

m_1	m_2	2
		+2
+1	+1	1

m_1	m_2	2	1
		+1	+1
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	2	1	0
		0	0	0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	2	1
		−1	−1
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	2
		−2
−1	−1	1

2×1/2

m_1	m_2	5/2
		+5/2
+2	+1/2	1

m_1	m_2	5/2	3/2
		+3/2	+3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

m_1	m_2	5/2	3/2
		+1/2	+1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	5/2	3/2
		−1/2	−1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	5/2	3/2
		−3/2	−3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

m_1	m_2	5/2
		−5/2
−2	−1/2	1

3/2×1/2

m_1	m_2	2
		+2
+3/2	+1/2	1

m_1	m_2	2	1
		+1	+1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

m_1	m_2	2	1
		0	0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	2	1
		−1	−1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	2
		−2
−3/2	−1/2	1

3/2×1

m_1	m_2	5/2
		+5/2
+3/2	+1	1

m_1	m_2	5/2	3/2
		+3/2	+3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

m_1	m_2	5/2	3/2	1/2
		+1/2	+1/2	+1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

m_1	m_2	5/2	3/2	1/2
		−1/2	−1/2	−1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

m_1	m_2	5/2	3/2
		−3/2	−3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

m_1	m_2	5/2
		−5/2
−3/2	−1	1

2×3/2

m_1	m_2	7/2
		+7/2
+2	+3/2	1

m_1	m_2	7/2	5/2
		+5/2	+5/2
+2	+1/2	3/7	4/7
+1	+3/2	4/7	−3/7

m_1	m_2	7/2	5/2	3/2
		+3/2	+3/2	+3/2
+2	−1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	−2/5
0	+3/2	2/7	−18/35	1/5

m_1	m_2	7/2	5/2	3/2	1/2
		+1/2	+1/2	+1/2	+1/2
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10

m_1	m_2	7/2	5/2	3/2	1/2
		−1/2	−1/2	−1/2	−1/2
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5

m_1	m_2	7/2	5/2	3/2
		−3/2	−3/2	−3/2
0	−3/2	2/7	18/35	1/5
−1	−1/2	4/7	−1/35	−2/5
−2	+1/2	1/7	−16/35	2/5

m_1	m_2	7/2	5/2
		−5/2	−5/2
−1	−3/2	4/7	3/7
−2	−1/2	3/7	−4/7

m_1	m_2	7/2
		−7/2
−2	−3/2	1

3/2×3/2

m_1	m_2	3
		+3
+3/2	+3/2	1

m_1	m_2	3	2
		+2	+2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

m_1	m_2	3	2	1
		+1	+1	+1
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

m_1	m_2	3	2	1	0
		0	0	0	0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

m_1	m_2	3	2	1
		−1	−1	−1
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

m_1	m_2	3	2
		−2	−2
−1/2	−3/2	1/2	1/2
−3/2	−1/2	1/2	−1/2

m_1	m_2	3
		−3
−3/2	−3/2	1

2×2

m_1	m_2	4
		+4
+2	+2	1

m_1	m_2	4	3
		+3	+3
+2	+1	1/2	1/2
+1	+2	1/2	−1/2

m_1	m_2	4	3	2
		+2	+2	+2
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	−3/7
0	+2	3/14	−1/2	2/7

m_1	m_2	4	3	2	1
		+1	+1	+1	+1
+2	−1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	−1/14	−3/10
0	+1	3/7	−1/5	−1/14	3/10
−1	+2	1/14	−3/10	3/7	−1/5

m_1	m_2	4	3	2	1	0
		0	0	0	0	0
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5

m_1	m_2	4	3	2	1
		−1	−1	−1	−1
+1	−2	1/14	3/10	3/7	1/5
0	−1	3/7	1/5	−1/14	−3/10
−1	0	3/7	−1/5	−1/14	3/10
−2	+1	1/14	−3/10	3/7	−1/5

m_1	m_2	4	3	2
		−2	−2	−2
0	−2	3/14	1/2	2/7
−1	−1	4/7	0	−3/7
−2	0	3/14	−1/2	2/7

m_1	m_2	4	3
		−3	−3
−1	−2	1/2	1/2
−2	−1	1/2	−1/2

m_1	m_2	4
		−4
−2	−2	1

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^1_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 35.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.