FYSH300 Particle physics
2. half-course exam (2. välikoe) 19.12.2012: 4 problems, **4 hours**. Return the **the question sheet** and particle tables together with your answer sheets – remember to write down your name in the problem sheet!

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1. a) What does one mean by *running* of the strong coupling, and by *asymptotic freedom* of QCD? (1p)

b) Explain, using Feynman diagrams, how one can probe the strong coupling constant α_s by measuring the decay width $\Gamma(\tau^- \to \nu_\tau + \text{hadrons})$. Indicate how the width $\Gamma(\tau^- \to \nu_\tau + \text{hadrons})$ depends on α_s. (1p)

c) The absolute values of the elements of the CKM-matrix have been measured to be as follows (ignoring the experimental error bars here):

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts|} & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97427 & 0.22534 & 0.00351 \\ 0.22520 & 0.97344 & 0.0412 \\ 0.00867 & 0.0404 & 0.999146 \end{pmatrix}$$

Using this, estimate the ratio of the following decay widths:

$$\frac{\Gamma(B^+(u\bar{b}) \to K^+(u\bar{s}) + \pi^0(u\bar{u}))}{\Gamma(B^+(u\bar{b}) \to \pi^+(u\bar{d}) + \overline{D^0}(u\bar{c}))}$$

where the quark content of the hadrons is shown in the parentheses, and where you don't have to think about any phase-space effects. Draw also the Feynman graphs for these processes. (2p)

d) Using lowest order QED perturbation theory, estimate the value of the ratio

$$R = \frac{\sigma(e^+e^- \to \ \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

in the region $\sqrt{s} = 4\ldots 8$ GeV in the cases of $N_C = 3$ and $N_C = 4$ colors. Draw the Feynman diagrams for these processes and indicate the coupling strengths in each vertex. Above the mass thresholds of the final state particles, you can assume an ultrarelativistic case, i.e. massless particles. [Note: this is a short calculation but answer, however, in sufficient details!] (2p)

2. Consider a theory whose Lagrange density is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2,$$

where ϕ is a complex scalar field which depends on the coordinate 4-vector x and which describes a charged spin-0 particle. The gauge field is A_μ and the field strength tensor is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This theory is invariant in local U(1) gauge (phase) transformations,

$$\phi \xrightarrow{U(x)} U(x)\phi, \quad \text{where} \quad U(x) = e^{i\alpha(x)}$$

where $\alpha(x)$ is real. The covariant derivative, $D_\mu = \partial_\mu - ieA_\mu$, is required to transform in these gauge transformations as

$$D_\mu\phi \xrightarrow{U(x)} U(x)D_\mu\phi.$$

a) Derive the transformation law for the gauge field A_μ in these gauge transformations. (1p)

b) Show, as briefly as possible, that $\phi^*\phi$ and $(D_\mu\phi)^*(D^\mu\phi)$, and $F_{\mu\nu}$ and thus also $F_{\mu\nu}F^{\mu\nu}$, are invariant in these gauge transformations. (1p)

c) Let's assume that $\lambda > 0$ but $\mu^2 < 0$, so that the Higgs mechanism is needed to find out the physical fields and masses of the theory. You don't have to do a detailed calculation here but explain the principle, i.e. write down how the fields $\phi(x)$ and $A_\mu(x)$, and the terms $\phi^*\phi$ and $(D_\mu\phi)^*(D^\mu\phi)$ and $F_{\mu\nu}F^{\mu\nu}$ transform in the Higgs mechanism. (1p)

d) After the Higgs mechanism, the Lagrangian of the broken-symmetry theory becomes

$$\tilde{\mathcal{L}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu h\partial^\mu h - \lambda v^2h^2 + \frac{1}{2}e^2v^2A_\mu A^\mu - \lambda vh^3 - \frac{1}{4}\lambda h^4 + \frac{1}{2}e^2A_\mu A^\mu h^2 + ve^2A_\mu A^\mu h$$

where $v^2 = -\mu^2/\lambda$ and $h = h(x)$ is the real scalar field.

i) Identify the mass terms and the particle masses in this theory (the factors of 2 you do not need to specify). (1p)

ii) Identify all the interaction terms of the broken-symmetry theory, and draw the vertices which each of these terms describes. In the figure, indicate the interaction strength in the vertex too. (1p)

iii) Draw all the tree-level (=non-loop) Feynman diagrams for the scattering

$$A + A \rightarrow h + h$$

which this broken-symmetry theory predicts. Indicate the coupling strengths in the graphs. (1p)

3. Let's consider the Standard Model (SM) Higgs particle (H^0) production and decays here.

a) One production channel through which the SM Higgs particle is searched for in the high-energy $p+p$ collisions at the LHC, is the heavy-vector-boson fusion with tagged jets (one in the forward direction and one in the backward direction). Draw a parton model example graph of such a SM Higgs production channel, and identify the colliding partons, beam jets, tagged jets and Higgs in the figure. (1p)

b) Write down, schematically, an expression for such a Higgs production cross section, $d\sigma(p+p \to H^0 + 2\,\mathrm{jets} + X)$, according to collinear factorization. (1p)

Consider then the following possible SM Higgs decay channels, marked in the figure on the next page (these are simulations for different Higgs masses):

c) Draw an example Feynman graph of the SM Higgs decay shown in each panel (draw these below each panel). Identify the particles and draw the arrows in your diagrams. (1p)

d) Into each panel, identify the Higgs signal particles and the following detector parts of the CMS experiment: Tracking chamber (TC), electromagnetic calorimeter (ECAL), hadron calorimeter (HCAL), muon detector (MD) (2p)

Let's then look at the recent measurements, shown in the two figures below.
Left: The invariant mass distribution of four leptons ($l^+l^-l^+l^-$)
Right: The invariant mass distribution of $e^{\pm}\mu^{\mp}$.

e) Explain why in the four-lepton channel there is a peak at $m_{llll} \approx 125$ GeV, while in the $e\mu$ channel such a peak is not seen at $m_{ll} \approx 125$ GeV. (1p)

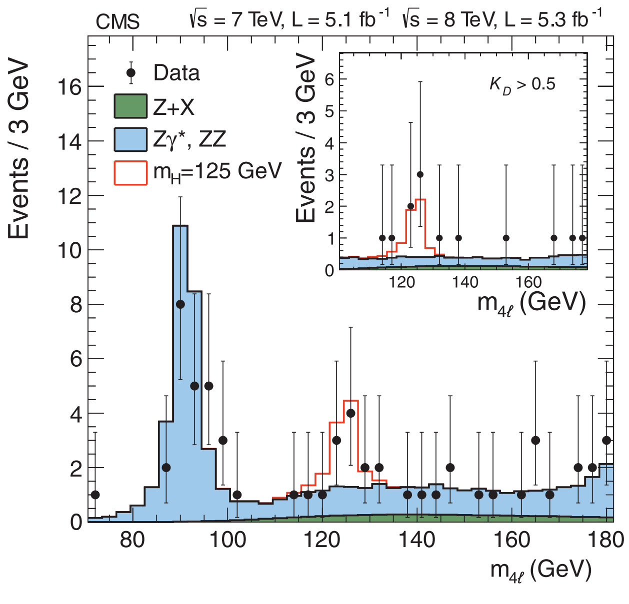


Fig. 4. Distribution of the four-lepton invariant mass for the $ZZ \to 4\ell$ analysis. The points represent the data, the filled histograms represent the background, and the open histogram shows the signal expectation for a Higgs boson of mass $m_H = 125$ GeV, added to the background expectation. The inset shows the $m_{4\ell}$ distribution after selection of events with $K_D > 0.5$, as described in the text.

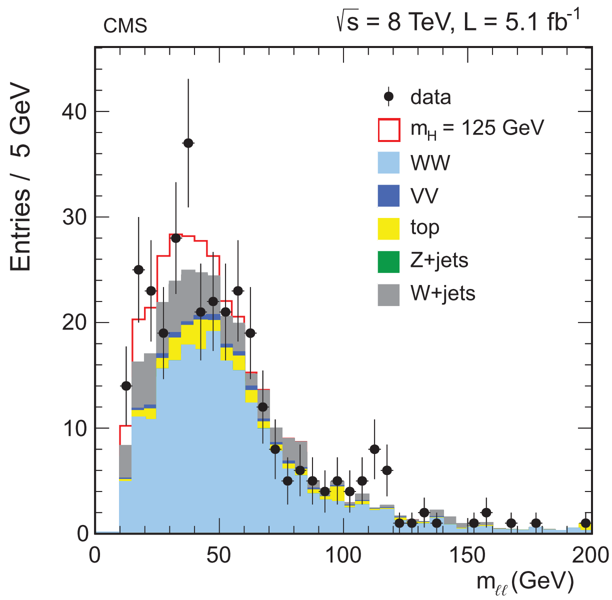
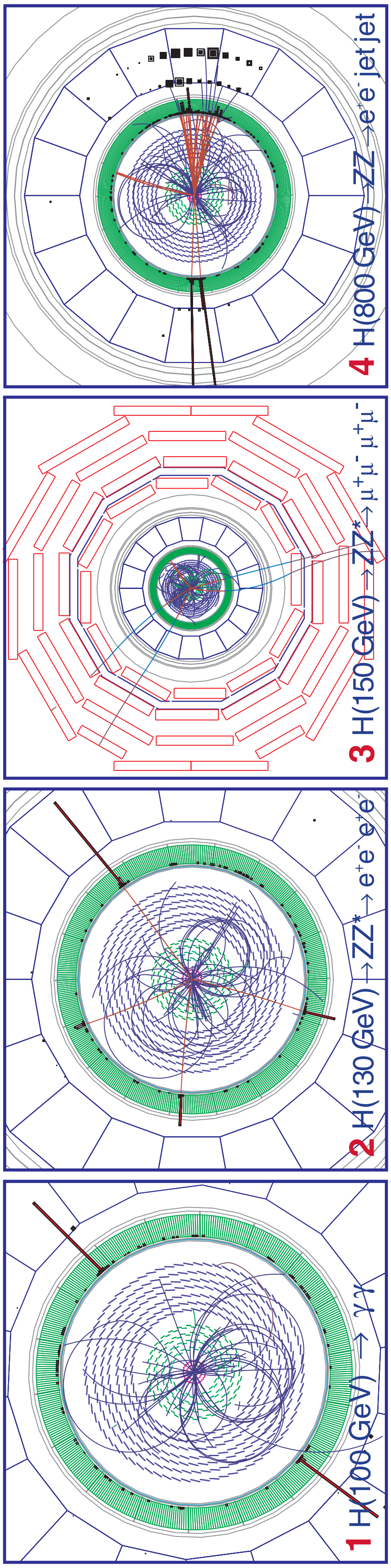


Fig. 7. Distribution of $m_{\ell\ell}$ for the zero-jet $e\mu$ category in the $H \to WW$ search at 8 TeV. The signal expected from a Higgs boson with a mass $m_H = 125$ GeV is shown added to the background.

YOUR NAME:



4 H(800 GeV) $\rightarrow$ ZZ $\rightarrow e^+e^-$ jet jet
3 H(150 GeV) $\rightarrow ZZ^* \rightarrow \mu^+\mu^-\mu^+\mu^-$
2 H(130 GeV) $\rightarrow ZZ^* \rightarrow e^+e^-e^+e^-$
1 H(100 GeV) $\rightarrow \gamma\gamma$

4. Consider the scattering

$$e^-(p_a) + \mu^-(p_b) \to e^-(p_c) + \mu^-(p_d)$$

at the ultrarelativistic limit, where the particle masses can be neglected, and in the leading order of the electromagnetic coupling. The 4-momenta of the particles are shown in the parentheses.

a) Using the Feynman rules of QED (see the attachment), compute the unpolarized differential cross section

$$\frac{d\sigma}{d\Omega_c^*} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s}$$

of this scattering and express the final result in terms of the Mandelstam variables $s = (p_a + p_b)^2$, $t = (p_a - p_c)^2$ and $u = (p_a - p_d)^2$. [*Hint:* Formulate the calculation in terms of the leptonic tensors $L^e_{\mu\nu}$ and $L^{\mu\nu}_{\mathrm{muon}}$. See the collection of formulae in the end of the paper for help in doing the spin summations.] (6p)

Bonus problem (extra +2p available) — do if you still have time and energy left!

b) Let's then suppose that instead of a spin-$\frac{1}{2}$ particle the muon is a spin-0 particle, keeping however the electron as a spin-$\frac{1}{2}$ particle. Compute the unpolarized differential cross section $\frac{d\sigma}{d\Omega_c^*}$ again. In this case the Feynman rule for the muon–photon vertex is not $-ie\gamma^\nu$ but $-ie(p_b + p_d)^\nu$, and the Feynman rule for the external muon legs is just 1. Express again the result in terms of the Mandelstam variables. [*Hint:* Before squaring the amplitude $\mathcal{M}$, use momentum conservation and the Dirac equations $\not{p}u(p) \approx 0$, $\bar{u}(p)\not{p} \approx 0$ to simplify the expression. Make use of the leptonic tensor you derived above.]

Collection of formulae

$$g_{\mu\nu} = g^{\mu\nu} \widehat{=} diag(1, -1, -1, -1)$$

$$A^\mu B_\mu = A^0 B^0 - \mathbf{A} \cdot \mathbf{B}$$

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\mathbf{1}_4$$

$$\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0$$

$$\gamma_\mu\gamma^\mu = 4\mathbf{1}_4$$

$$\gamma_\mu \not{a} \gamma^\mu = -2\not{a}, \text{ where } \not{a} \equiv \gamma_\mu a^\mu$$

$$\gamma_\mu \not{a} \not{b} \gamma^\mu = 4a \cdot b$$

$$\gamma_\mu \not{a} \not{b} \not{c} \gamma^\mu = -2\not{c} \not{b} \not{a}$$

$$\gamma^{5\dagger} = \gamma^5 \text{ , where } \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$(\gamma^5)^2 = \mathbf{1}_4$$

$$\{\gamma^5, \gamma^\mu\} = 0$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})$$

$$\mathrm{Tr}(\gamma^5) = 0$$

$$\mathrm{Tr}(\gamma^{\mu_1}\gamma^{\mu_2} \dots \gamma^{\mu_{2n+1}}) = 0$$

Projection operators for Dirac spinors:

$$\sum_{s=1,2} u^{(s)}(p)\overline{u}^{(s)}(p) = \not{p} + m \qquad \sum_{s=1,2} v^{(s)}(p)\overline{v}^{(s)}(p) = \not{p} - m,$$

where $\overline{u} \equiv u^\dagger\gamma^0$ and $\overline{v} \equiv v^\dagger\gamma^0$

Dirac equations: $(\not{p} - m)u(p) = 0 \qquad (\not{p} + m)v(p) = 0$

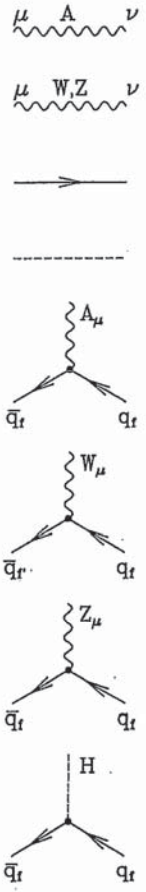
Cross section $ab \to cd$ (when $m_{a,b} = m_{c,d}$):

$$\frac{d\sigma}{d\Omega^*} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s}$$

Spherical coordinates:

$$\int d\Omega = \int_0^{2\pi} d\phi \int_{-1}^{1} d(\cos\theta)$$

⇒ <u>Feynmanin säännöt</u> (ks. esim. Ellis, Stirling, Webber, QCD an Collider Physics)



$[-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}]\frac{i}{q^2}$

$[-g^{\mu\nu}+\frac{q^\mu q^\nu}{M^2}]\frac{i}{(q^2-M^2)}$

h nom. massa

$\frac{i}{\not{q}-m}$

$\frac{i}{(q^2-M_H^2)}$

$-ieQ_f\gamma_\mu$

$=\gamma^5$ $\quad T^+=\begin{pmatrix}0&1\\0&0\end{pmatrix}$

$\frac{-ig_W}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)(T^+)_{f'f}$ $\quad \underset{=g}{g_W}=\frac{e}{\sin\theta_W}$

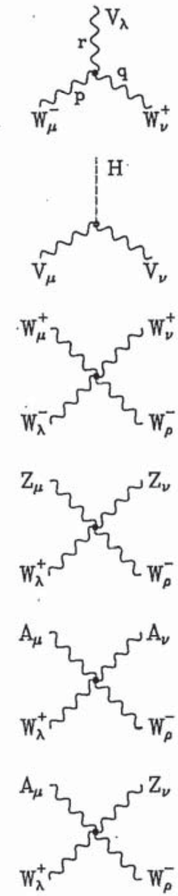
$\frac{-ig_W}{2\cos\theta_W}\gamma^\mu(V_f-A_f\gamma_5)$

($V_f=T_f^3-2Q_f\sin^2\theta_W$, $A_f=T_f^3$)

	Q_f	V_f	A_f
$q_{ylä}$	$\frac{2}{3}$	$+\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$	$+\frac{1}{2}$
q_{ala}	$-\frac{1}{3}$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}$
ν_l	0	$+\frac{1}{2}$	$+\frac{1}{2}$
l_l	-1	$-\frac{1}{2}+2\sin^2\theta_W$	$-\frac{1}{2}$

$\frac{-ig_W m_f}{2M_W}$

Fig. 8.5. Propagators and Feynman rules for fermion interactions.



$+ig_V\,[(p-q)_\lambda g_{\mu\nu}+(q-r)_\mu g_{\nu\lambda}+(r-p)_\nu g_{\lambda\mu}]$

(all momenta incoming, $g_A=e$, $g_Z=g_W\cos\theta_W$)

$g_W=e/\sin\theta_W$

$+ig_{VH}M_W g_{\mu\nu}$

($g_{WH}=g_W$, $g_{ZH}=g_W/\cos^2\theta_W$)

$+ig_W^2[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$

$-ig_W^2\cos^2\theta_W[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$

$-ie^2[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$

$-ieg_W\cos\theta_W[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$

Fig. 8.2. Feynman rules for boson interactions.

Field: Applications of perturbative QCD

C.1 Feynman Rules – General Discussion

External Lines:
For particles in the initial or final state one writes the following factor:

(a) Spin Zero Boson 1

QED → fotoni (b) Spin One Boson .... $\epsilon_\mu(\lambda)$

Here $\epsilon_\mu(\lambda)$ is the polarization 4-vector for a boson with helicity λ. For the case of a massless spin one boson propagating along the $\hat{z}$-axis with 4-momentum k_μ given by

$$k_\mu = \begin{pmatrix} k_0 \\ 0 \\ 0 \\ k_3 \end{pmatrix} \quad , \tag{C.1.1}$$

with $k_0 = |k_3|$, the polarization 4-vectors are given by

$$\epsilon_\mu(\lambda = \pm 1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix} \quad , \tag{C.1.2}$$

for helicity ± 1. The polarization 4-vectors satisfy

$$k \cdot \epsilon = 0, \tag{C.1.3}$$

and

$$\epsilon^2 = -1. \tag{C.1.4}$$

For a massive spin 1 boson with 4-momentum given by (C.1.1) but with $k_0^2 = k_3^2 + M^2$ one also has the longitudinal state

$$\epsilon_\mu(\lambda = 0) = \frac{1}{M} \begin{pmatrix} k_3 \\ 0 \\ 0 \\ k_0 \end{pmatrix} \quad , \tag{C.1.5}$$

where M is the boson mass.

QED → (c) Spin 1/2 fermion of momentum p and spin s $\leftarrow \equiv S_z$

in initial state .... $u(p, s)$ on the right

in final state .... $\bar{u}(p, s)$ on the left

QED → (d) Spin 1/2 antifermion of momentum p and spin s

in initial state .... $\bar{v}(p, s)$ on the left

in final state .... $v(p, s)$ on the right

Internal Lines (Propagators):
Each internal line describes a particle of momentum q and mass m. Some examples are as follows: $(\varepsilon \to 0)$

(a) Spin Zero Boson

$$\frac{i}{q^2 - m^2 + i\epsilon} \tag{C.1.6}$$

QED → (b) Photon (Feynman Gauge)

$$\frac{-ig_{\mu\nu}}{q^2 + i\epsilon} \tag{C.1.7}$$

(c) Spin One Boson

$$\frac{-i\left(g_{\mu\nu} - q_\mu q_\nu / m^2\right)}{q^2 - m^2 + i\epsilon} \tag{C.1.8}$$

QED → (d) Spin 1/2 fermion

$$\frac{i(\not{q} + m)}{q^2 - m^2 + i\epsilon} \tag{C.1.9}$$

For antifermions one uses the same propagator, treating the antifermion as a fermion of opposite 4-momentum (*i.e.*, $-q$).

Vertex Factors:
For each intersection of three (or more) lines at one point there is a vertex factor which depends on the structure of the interaction Lagrangian. Some examples are shown in Fig. C.1.

(a) Three scalar boson vertex

$$-ig \tag{C.1.10}$$

QED → (b) $ee\gamma$ vertex

$$-ie\gamma_\mu \tag{C.1.11}$$

Here e is the charge of the electron and the fine structure constant $\alpha = e^2/(4\pi)$.

(c) Charged spin zero boson-photon vertex

$$-iQ(p_1 + p_2)_\mu \tag{C.1.12}$$

(d) Four point coupling for charged spin zero boson-photon

$$2iQ^2 g_{\mu\nu} \tag{C.1.13}$$

Loops and Combinatorics:

(a) For each loop with undetermined momentum k .... $\int d^4k/(2\pi)^4$
Here the integral runs over all values of the momentum.

(b) For each closed fermion loop .... -1

(c) For each closed loop containing n identical bosons ...... $1/n!$

-292-