FYSH300 Particle Physics 30.8.2013

Final Exam, 5 problems, 4 hours. Return the tables together with the answer sheets.

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1. Lets's consider production of the Standard Model Higgs particle in $p+p$ collisions at the CERN-LHC.

i) Explain briefly, in your own words, how the Higgs particle "creates" masses for the massive particles in the Standard Model. *(1/6p)*

ii) Draw a detailed parton-model Feynman graph, where the Higgs particle is created and where it decays into two photons, i.e. a Feynman graph of the inclusive process

$$p+p \to H+X \to \gamma\gamma+X.$$

Mark in your graph also the particle/momentum arrows, identity of all the relevant particles, and indicate also the unidentified part X. In thinking about the Higgs particle couplings to other particles, you may find help in the attached table of the Feynman rules of the Standard Model. *(1p)*

iii) Draw also one Feynman graph (detailed graph, according to the parton model) of a background process for the Higgs production process which we considered above, i.e. of such a process, where the final state is identical to 2 photons+X. *(1p)*

iv) Sketch a figure of the cross section of the above process $p+p \to \gamma\gamma+X$ as a function of the invariant mass of the photon pair, in the mass range 115— 150 GeV, taking into account that the Higgs mass is 125 GeV. In your figure, indicate where we can see the Higgs signal and where is the background, and give also a definition to the invariant mass of the photon pair.

(1p)

v) The CMS and ATLAS experiments at CERN have now (most likely) found the Higgs particle by using a detector which measures the energy of hadrons, electrons and photons, identifies the muons and tracks charged particles. Sketch a picture of a detector which could do these things. In your figure, identify (i.e. name) each part of the detector. *(1p)*

vi) Let's consider the following decay channel of the Higgs particle:

$$H \to ZZ \to (e^- + e^+) + (\text{jet} + \text{jet}).$$

Draw, in the detector picture you scetched above, what happens to the observed final state leptons and jets, when they propagate in your detector – i.e. sketch a picture of how such a final state would look like in your detector (and if nothing happens in some part of the detector when the particle is passing through it, draw a dashed line).

(1p)

2. Let's consider here an elastic process $e{+}p \to e{+}p$ in the colliding beams experiments at the DESY-HERA accelerator, where the energy of the electron was $E_1 = 30$ GeV and the energy of the proton $E_2 = 920$ GeV. In this ultrarelativistic case $m_e, m_p \ll E_1, E_2$, so that you can set the particle masses here to zero.

a) What is the CMS energy of these collisions? *(1p)*

Suppose then that in the CMS frame we observe that the electron scattered into an angle $\theta_3^* = 60°$, i.e. that $\cos\theta_3^* = +\frac{1}{2}$.

b) Sketch a figure of the momentum vectors and scattering angles in this scattering

i) in the CMS frame, and

ii) in the collider frame where the energies of the beams are the E_1 and E_2 mentioned above.

(1p)

c) Starting from the definitions of the Mandelstam variables s, t and u, compute the scattering angle θ_3 and the energy E_3 of the scattered electron in the collider frame where the energies of the beams are the E_1 and E_2 mentioned above. Express $\cos\theta_3$ and E_3 in terms of the energies E_1 and E_2 and compute also the numerical values of θ_3 and E_3. *(4p)*

[Recall that with our choice of metrics, the scalar product of two 4-vectors is $A^\mu B_\mu = A^0 B^0 - \mathbf{A} \cdot \mathbf{B}$]

3. Let's consider the decays of the $\eta(547)$ meson (which is a $q\bar{q}$ state). Suppose that we know the branching ratios into the following decay channels:

(i) $B(\eta \to \gamma + \gamma) = 39$ %

(ii) $B(\eta \to \pi^0 + \pi^0 + \pi^0) = 33$ %

and that the decay channel $\eta \to \gamma + \gamma + \gamma$ has <u>not</u> been discovered.

a) Through which interaction does the decay (ii) here proceed – and why? *(1/6p)*

b) Figure out the C parity of the photon and the η meson. *(1/6p)*

c) In addition, we know that η is an isospin singlet state $|I = 0, I_3 = 0\rangle$. Show that the decay $\eta \to \pi^0 + \pi^0 + \pi^0$ through the strong interaction would break the isospin conservation. *Hints:* π^0 is a $|I = 1, I_3 = 0\rangle$ state. Couple first two π^0s together, and then the third π^0 with them. Make use of the attached table for the CG coefficients. *(4/6p)*

4. Let's consider an $SU(N)$ symmetric non-Abelian gauge-field theory, whose Lagrange density is

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} + i\overline{\Psi}\gamma^\mu D_\mu \Psi - m\overline{\Psi}\Psi$$

where $m > 0$ and Ψ is a complex spinor which describes the spin-$\frac{1}{2}$ fermion of this theory. The fermion mass is m, and $\overline{\Psi} = \Psi^\dagger \gamma^0$. Let's call this particle F and its antiparticle $\overline{F}$. The gauge fields A^a_μ describe the gauge bosons of this theory, let's call these G. The field strength tensor is now

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \lambda f^{abc} A^b_\mu A^c_\nu,$$

where λ is the coupling constant of this theory and the coefficients f^{abc} are the structure constants of the group $SU(N)$. The covariant derivative is here $D_\mu = \partial_\mu + i\lambda t^a A^a_\mu$, where t^a are the generator matrices of this group.

a) Identify carefully the kinetic terms, the mass terms and the interaction terms of this theory. Draw the basic vertex for each interaction term, and indicate the particle content and the dependence on λ of each vertex. *(3/6p)*

b) Draw all Feynman graphs for the following scatterings, according to the above theory, in the lowest possible order in the coupling λ:

$$F + \overline{F} \to F + \overline{F},$$

$$G + G \to G + G,$$

$$F + \overline{F} \to F + \overline{F} + G$$

On the basis of the graphs you drew, figure out how the invariant amplitudes $\mathcal{M}$ and the cross sections of the above processes depend on λ. [*Note:* Your answer here should be based only on the basic vertices you found in the item a) above, so please be careful with the item a)!] *(3/6p)*

5. The figure below describes the measurement of the angular distribution (in the CMS frame) of muons produced in the unpolarized scattering

$$e^+ + e^- \to \mu^+ + \mu^-$$

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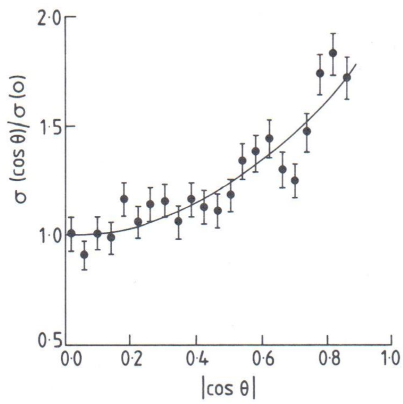


Starting from the Feynman rules, compute the differential cross section $d\sigma/d\cos\theta^*$ of this process, in the lowest order in the electromagnetic interaction. Express the final result in terms of the scattering angle θ^* in the CMS-frame, fine-structure constant $\alpha = \frac{e^2}{4\pi}$ and the CMS energy $\sqrt{s}$, assuming that $\sqrt{s}$ is in the range 15—40 GeV. After this, form the quantity

$$\frac{\frac{d\sigma}{d\cos\theta^*}}{\frac{d\sigma}{d\cos\theta^*}|_{\cos\theta^*=0}},$$

which is shown in the figure and compare your result with the figure. In the end, explain briefly what one can learn from measuring such an angular distribution.

Instructions: Let's consider here only the high-energy limit, i.e. you can set the particle masses in the initial and final state to zero. Please use the following notation: p_a for the 4-momentum of the positron, p_b for the 4-momentum of the electron, p_c for the antimuon and p_d for the muon. Mark all the intermediate steps you perform in your answer sheet – and not on a scrap paper. Make use of the attached table (Field C.1) of the Feynman rules and the collection of the formulae.

Collection of formulae

$$g_{\mu\nu} = g^{\mu\nu} \hat{=} diag(1, -1, -1, -1)$$

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\mathbf{1}_4$$

$$\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0$$

$$\gamma_\mu\gamma^\mu = 4\mathbf{1}_4$$

$$\gamma_\mu \not{a} \gamma^\mu = -2\not{a}, \text{ where } \not{a} \equiv \gamma_\mu a^\mu$$

$$\gamma_\mu \not{a} \not{b} \gamma^\mu = 4a \cdot b$$

$$\gamma_\mu \not{a} \not{b} \not{c} \gamma^\mu = -2\not{c} \not{b} \not{a}$$

$$\gamma^{5\dagger} = \gamma^5 \text{ , where } \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$(\gamma^5)^2 = \mathbf{1}_4$$

$$\{\gamma^5, \gamma^\mu\} = 0$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})$$

$$\mathrm{Tr}(\gamma^5) = 0$$

$$\mathrm{Tr}(\gamma^{\mu_1}\gamma^{\mu_2}\ldots\gamma^{\mu_{2n+1}}) = 0$$

Projection operators for Dirac spinors:

$$\sum_{s=1,2} u^{(s)}(p)\overline{u}^{(s)}(p) = \not{p} + m \qquad \sum_{s=1,2} v^{(s)}(p)\overline{v}^{(s)}(p) = \not{p} - m,$$

where $\overline{u} \equiv u^\dagger\gamma^0$ and $\overline{v} \equiv v^\dagger\gamma^0$

Dirac equation: $(\not{p} - m)u(p) = 0 \qquad (\not{p} + m)v(p) = 0$

Cross section $ab \to cd$ (when $m_{a,b} = m_{c,d}$):

$$\frac{d\sigma}{d\Omega^*} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s}$$

Spherical coordinates:

$$\int d\Omega = \int_0^{2\pi} d\phi \int_{-1}^{1} d(\cos\theta)$$

⇒ <u>Feynmanin säännöt</u> (ks. esim. Ellis, Stirling, Webber, QCD an Collider Physics)

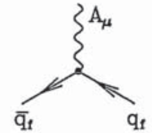
$$[-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}]\frac{i}{q^2}$$

$$[-g^{\mu\nu}+\frac{q^\mu q^\nu}{M^2}]\frac{i}{(q^2-M^2)}$$

h nom. massa

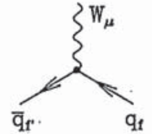
$$\frac{i}{\not{q}-m}$$

$$\frac{i}{(q^2-M_H^2)}$$

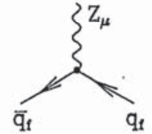


$$-ieQ_f\gamma_\mu$$

$= \gamma^5$ $\quad T^+=\begin{pmatrix}0&1\\0&0\end{pmatrix}$



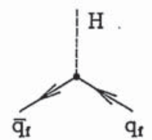
$$\frac{-ig_W}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)(T^+)_{f'f} \qquad \underbrace{g_W}_{=g}=\frac{e}{\sin\theta_W}$$



$$\frac{-ig_W}{2\cos\theta_W}\gamma^\mu(V_f-A_f\gamma_5)$$

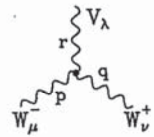
$(V_f=T_f^3-2Q_f\sin^2\theta_W,\quad A_f=T_f^3)$

	Q_f	V_f	A_f
$q_{ylä}$	$\frac{2}{3}$	$+\frac{1}{2}-\frac{4}{3}\sin^2\theta_w$	$+\frac{1}{2}$
q_{ala}	$-\frac{1}{3}$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_w$	$-\frac{1}{2}$
ν_ℓ	0	$+\frac{1}{2}$	$+\frac{1}{2}$
ℓ_ℓ	-1	$-\frac{1}{2}+2\sin^2\theta_w$	$-\frac{1}{2}$



$$\frac{-ig_W m_f}{2M_W}$$

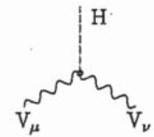
Fig. 8.5. Propagators and Feynman rules for fermion interactions.



$$+ig_V\ [(p-q)_\lambda g_{\mu\nu}+(q-r)_\mu g_{\nu\lambda}+(r-p)_\nu g_{\lambda\mu}]$$

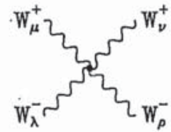
(all momenta incoming, $g_A=e,\ g_Z=g_W\cos\theta_W$)

$g_W = e/\sin\theta_W$

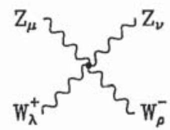


$$+ig_{VH}M_W g_{\mu\nu}$$

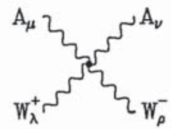
$(g_{WH}=g_W,\ g_{ZH}=g_W/\cos^2\theta_W)$



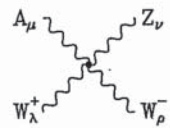
$$+ig_W^2[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$



$$-ig_W^2\cos^2\theta_W[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$



$$-ie^2[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$



$$-ieg_W\cos\theta_W[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$

Fig. 8.2. Feynman rules for boson interactions.

35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2	Coefficients		
m_1	m_2			
.	.			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^{\,\ell}_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

1/2×1/2

m_1	m_2	J=1, M=+1
+1/2	+1/2	1

m_1	m_2	J=1, M=0	J=0, M=0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	J=1, M=−1
−1/2	−1/2	1

1×1/2

m_1	m_2	3/2, +3/2
+1	+1/2	1

m_1	m_2	3/2, +1/2	1/2, +1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	3/2, −1/2	1/2, −1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	3/2, −3/2
−1	−1/2	1

2×1/2

m_1	m_2	5/2, +5/2
+2	+1/2	1

m_1	m_2	5/2, +3/2	3/2, +3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

m_1	m_2	5/2, +1/2	3/2, +1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	5/2, −1/2	3/2, −1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	5/2, −3/2	3/2, −3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

m_1	m_2	5/2, −5/2
−2	−1/2	1

3/2×1/2

m_1	m_2	2, +2
+3/2	+1/2	1

m_1	m_2	2, +1	1, +1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

m_1	m_2	2, 0	1, 0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	2, −1	1, −1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	2, −2
−3/2	−1/2	1

2×1

m_1	m_2	3, +3
+2	+1	1

m_1	m_2	3, +2	2, +2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	3, +1	2, +1	1, +1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

m_1	m_2	3, 0	2, 0	1, 0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	3, −1	2, −1	1, −1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	3, −2	2, −2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	3, −3
−2	−1	1

3/2×1

m_1	m_2	5/2, +5/2
+3/2	+1	1

m_1	m_2	5/2, +3/2	3/2, +3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

m_1	m_2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

m_1	m_2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

m_1	m_2	5/2, −3/2	3/2, −3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

m_1	m_2	5/2, −5/2
−3/2	−1	1

1×1

m_1	m_2	2, +2
+1	+1	1

m_1	m_2	2, +1	1, +1
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	2, 0	1, 0	0, 0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	2, −1	1, −1
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	2, −2
−1	−1	1

$$d^{\,j}_{m',m} = (-1)^{m-m'} d^{\,j}_{m,m'} = d^{\,j}_{-m,-m'}$$

2×3/2

m_1	m_2	7/2, +7/2
+2	+3/2	1

m_1	m_2	7/2, +5/2	5/2, +5/2
+2	+1/2	3/7	4/7
+1	+3/2	4/7	−3/7

m_1	m_2	7/2, +3/2	5/2, +3/2	3/2, +3/2
+2	−1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	−2/5
0	+3/2	2/7	−18/35	1/5

m_1	m_2	7/2, +1/2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10

m_1	m_2	7/2, −1/2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5

m_1	m_2	7/2, −3/2	5/2, −3/2	3/2, −3/2
0	−3/2	2/7	18/35	1/5
−1	−1/2	4/7	−1/35	−2/5
−2	+1/2	1/7	−16/35	2/5

m_1	m_2	7/2, −5/2	5/2, −5/2
−1	−3/2	4/7	3/7
−2	−1/2	3/7	−4/7

m_1	m_2	7/2, −7/2
−2	−3/2	1

3/2×3/2

m_1	m_2	3, +3
+3/2	+3/2	1

m_1	m_2	3, +2	2, +2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

m_1	m_2	3, +1	2, +1	1, +1
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

m_1	m_2	3, 0	2, 0	1, 0	0, 0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

m_1	m_2	3, −1	2, −1	1, −1
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

m_1	m_2	3, −2	2, −2
−1/2	−3/2	1/2	1/2
−3/2	−1/2	1/2	−1/2

m_1	m_2	3, −3
−3/2	−3/2	1

$$d^{\,1}_{0,0} = \cos\theta \qquad d^{\,1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{\,1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^{\,1}_{1,1} = \frac{1+\cos\theta}{2} \qquad d^{\,1}_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^{\,1}_{1,-1} = \frac{1-\cos\theta}{2}$$

2×2

m_1	m_2	4, +4
+2	+2	1

m_1	m_2	4, +3	3, +3
+2	+1	1/2	1/2
+1	+2	1/2	−1/2

m_1	m_2	4, +2	3, +2	2, +2
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	−3/7
0	+2	3/14	−1/2	2/7

m_1	m_2	4, +1	3, +1	2, +1	1, +1
+2	−1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	−1/14	−3/10
0	+1	3/7	−1/5	−1/14	3/10
−1	+2	1/14	−3/10	3/7	−1/5

m_1	m_2	4, 0	3, 0	2, 0	1, 0	0, 0
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5

m_1	m_2	4, −1	3, −1	2, −1	1, −1
+1	−2	1/14	3/10	3/7	1/5
0	−1	3/7	1/5	−1/14	−3/10
−1	0	3/7	−1/5	−1/14	3/10
−2	+1	1/14	−3/10	3/7	−1/5

m_1	m_2	4, −2	3, −2	2, −2
0	−2	3/14	1/2	2/7
−1	−1	4/7	0	−3/7
−2	0	3/14	−1/2	2/7

m_1	m_2	4, −3	3, −3
−1	−2	1/2	1/2
−2	−1	1/2	−1/2

m_1	m_2	4, −4
−2	−2	1

$$d^{\,3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{\,3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^{\,2}_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^{\,2}_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{\,2}_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^{\,2}_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^{\,2}_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^{\,2}_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{\,2}_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{\,2}_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^{\,2}_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 35.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.

Field: Applications of perturbative QCD

C.1 Feynman Rules – General Discussion

External Lines:

For particles in the initial or final state one writes the following factor:

(a) Spin Zero Boson 1

QED → fotoni (b) Spin One Boson .... $\epsilon_\mu(\lambda)$

Here $\epsilon_\mu(\lambda)$ is the polarization 4-vector for a boson with helicity λ. For the case of a massless spin one boson propagating along the $\hat{z}$-axis with 4-momentum k_μ given by

$$k_\mu = \begin{pmatrix} k_0 \\ 0 \\ 0 \\ k_3 \end{pmatrix} \quad , \tag{C.1.1}$$

with $k_0 = |k_3|$, the polarization 4-vectors are given by

$$\epsilon_\mu(\lambda = \pm 1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix} \quad , \tag{C.1.2}$$

for helicity ± 1. The polarization 4-vectors satisfy

$$k \cdot \epsilon = 0, \tag{C.1.3}$$

and

$$\epsilon^2 = -1. \tag{C.1.4}$$

For a massive spin 1 boson with 4-momentum given by (C.1.1) but with $k_0^2 = k_3^2 + M^2$ one also has the longitudinal state

$$\epsilon_\mu(\lambda = 0) = \frac{1}{M} \begin{pmatrix} k_3 \\ 0 \\ 0 \\ k_0 \end{pmatrix} \quad , \tag{C.1.5}$$

where M is the boson mass.

QED → (c) Spin 1/2 fermion of momentum p and spin s $\equiv S_z$

in initial state .... $u(p, s)$ on the right

in final state .... $\bar{u}(p, s)$ on the left

QED → (d) Spin 1/2 antifermion of momentum p and spin s

in initial state .... $\bar{v}(p, s)$ on the left

in final state .... $v(p, s)$ on the right

Internal Lines (Propagators):

Each internal line describes a particle of momentum q and mass m. Some examples are as follows: $(\varepsilon \to 0)$

(a) Spin Zero Boson

$$\frac{i}{q^2 - m^2 + i\epsilon} \tag{C.1.6}$$

QED → (b) Photon (Feynman Gauge)

$$\frac{-ig_{\mu\nu}}{q^2 + i\epsilon} \tag{C.1.7}$$

(c) Spin One Boson

$$\frac{-i\,(g_{\mu\nu} - q_\mu q_\nu/m^2)}{q^2 - m^2 + i\epsilon} \tag{C.1.8}$$

QED → (d) Spin 1/2 fermion

$$\frac{i(\not{q} + m)}{q^2 - m^2 + i\epsilon} \tag{C.1.9}$$

For antifermions one uses the same propagator, treating the antifermion as a fermion of opposite 4-momentum (*i.e.*, $-q$).

Vertex Factors:

For each intersection of three (or more) lines at one point there is a vertex factor which depends on the structure of the interaction Lagrangian. Some examples are shown in Fig. C.1.

(a) Three scalar boson vertex

$$-ig \tag{C.1.10}$$

QED → (b) $ee\gamma$ vertex

$$-ie\gamma_\mu \tag{C.1.11}$$

Here e is the charge of the electron and the fine structure constant $\alpha = e^2/(4\pi)$.

(c) Charged spin zero boson-photon vertex

$$-iQ(p_1 + p_2)_\mu \tag{C.1.12}$$

(d) Four point coupling for charged spin zero boson-photon

$$2iQ^2 g_{\mu\nu} \tag{C.1.13}$$

Loops and Combinatorics:

(a) For each loop with undetermined momentum k .... $\int d^4k/(2\pi)^4$

Here the integral runs over all values of the momentum.

(b) For each closed fermion loop .... -1

(c) For each closed loop containing n identical bosons ...... $1/n!$

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