FYSH300 fall 2011

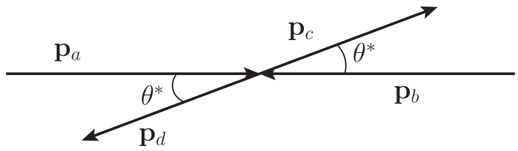
Final exam Friday June 15, 2012. Loppukoe pe 15.6.2012

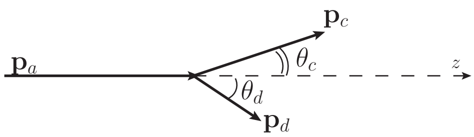
Time: 4 hours

Answer in Finnish or English. Vastaa valintasi mukaan suomeksi tai englanniksi.

1. Kinematics

   Consider an elastic antiproton-proton collision. Assume that we have a collision where in the CMS frame we have $\theta^* = 60°$ and $|\mathbf{p}^*| = \sqrt{3}m_p$. Calculate the energy of the outgoing antiproton in the target (proton) rest frame E_c^{TRF}.





2. Interactions

   The following reactions are *not* possible, at least in the standard model, why?

   (a) (1p) $\mu^- + e^- \to \nu_e + \nu_\mu$
   (b) (1p) $K^- + p \to \Sigma^+ + n + \pi^-$
   (c) (1p) $\Sigma^- \to n + e^- + \nu_e$

   The following reactions are possible. What interactions cause them? (If they can happen through different interactions, name the strongest/most likely one.) Draw an appropriate Feynman diagram or quark diagram.

   (d) (1p) $\mu^- + e^+ \to \bar{\nu}_e + \nu_\mu$
   (e) (1p) $\pi^0 \to 2\gamma$
   (f) (1p) $\pi^- + p \to \pi^0 + n$

3. Masses

   (a) (1p) Why are mass terms absent from the electroweak Lagrangean before symmetry breaking?
   (b) (1p) How does a lepton mass arise in the Standard Model? Sketch the relevant Yukawa term in the Lagrangean.
   (c) (1p) How do $W^\pm$ masses arise in the Standard Model? Sketch the relevant term.
   (d) (1p) What makes the proton mass in the Standard Model?
   (e) (1p) What is the difference between mass eigenstates and flavour eigenstates?
   (f) (1p) Explain and give examples for Cabibbo-suppressed and Cabibbo-allowed processes.

4. Quantum Electrodynamics (QED)

   (a) (2p) Show that the Dirac Langrangean

   $$\mathcal{L}_D = i\overline{\Psi}\gamma^\mu\partial_\mu\Psi - m\overline{\Psi}\Psi$$

   is invariant under global $U(1)$ rotations $e^{i\alpha}$ of the spinor fields but not under local rotations $e^{i\alpha(x)}$ unless an additional field is introduced into the Lagrangean. Argue why this field must be a vector field.

   (b) (2p) Show that the introduction of the covariant derivative $D_\mu = \partial_\mu + ieA_\mu$ and a vector field kinetic term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ makes the Lagrangean gauge invariant if the vector field transforms as $A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\alpha(x)$.

   (c) (1p) In what sense is A_μ a physical quantity? How is it similar to the QCD gluon, how is it different?

   (d) (1p) If we know the quark charges, how does $R = \frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)}$ measure the number of colors in QCD? Sketch the QED process.

5. Symmetries

(a) (2p) What is the role of broken symmetries in the Standard Model? Explain dynamical symmetry breaking with the example of a complex scalar field with the potential $V(x) = m^2\phi^*\phi + \lambda|\phi * \phi|^2$.

(b) (2p) Explain the role of C and P symmetry in the standard model.

(c) (2p) We know that the η meson is an isospin singlet $|I, I_3\rangle = |0, 0\rangle$ and the pions form a triplet $-|\pi^+\rangle, |\pi_0\rangle, |\pi^-\rangle = |1, 1\rangle, |1, 0\rangle, |1, -1\rangle$. Does isospin symmetry permit the decay $\eta \to \pi^0 + \pi^0 + \pi^0$?

Useful expressions:
The Pauli spin matrices are

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The Dirac matrices are

$$\alpha^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} \mathbf{1_2} & 0 \\ 0 & -\mathbf{1_2} \end{pmatrix}$$

The relations between the physical electroweak gauge Bosons $W_\mu^\pm, Z_\mu$ and A_μ and the gauge fields appearing in the primary Lagrangean A_μ^i, B_μ before symmetry breaking are

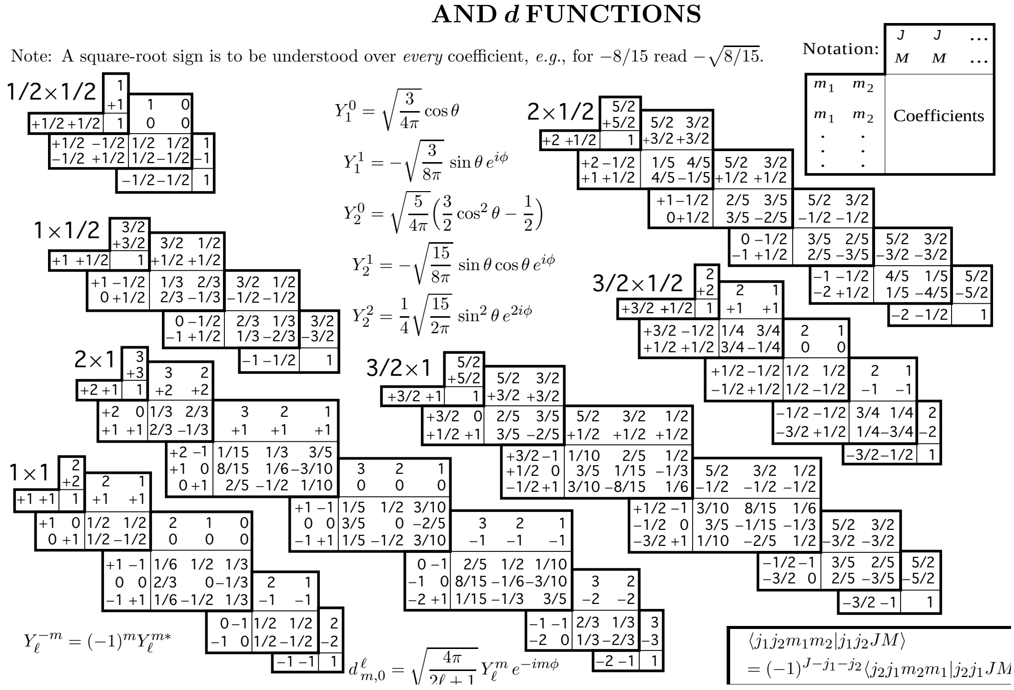
$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2)$$

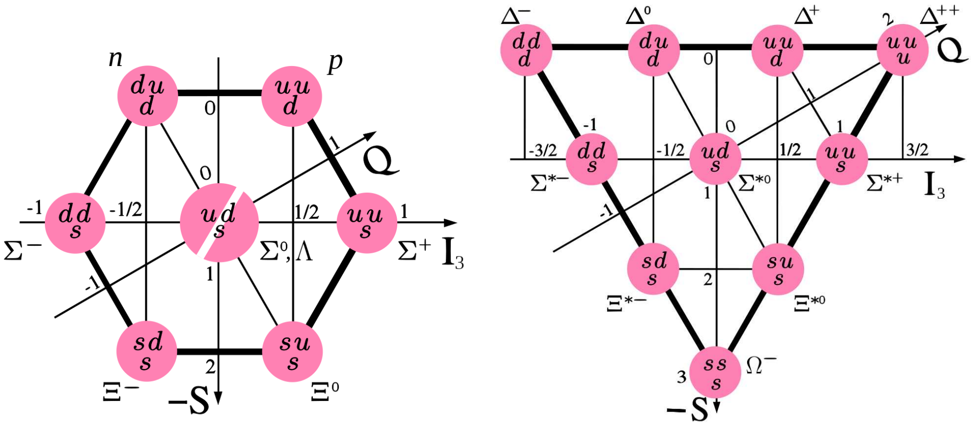
and

$$\begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

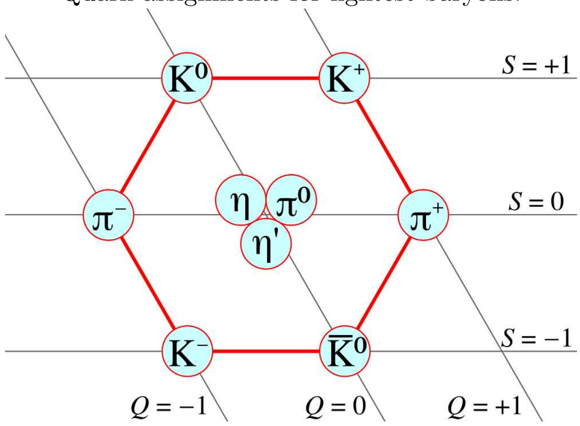
with the Weinberg angle θ_W.

36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONIC AND *d* FUNCTIONS





Quark assignments for lightest baryons.



Lightest meson nonet (pseudoscalar mesons consisting of one u, d or s quark and one $\bar{u}, \bar{d}$ or $\bar{s}$ antiquark.) Reminder: strange quark has $S = -1$. From S and the electric charge Q of the meson you can reconstruct the quark content.