FYSH300 fall 2013

Midterm exam Friday November 15, 2013. Time: 4 hours. Välikoe pe 15.11.2013. Aikaa 4 tuntia.
Answer in Finnish or English. Vastaa valintasi mukaan suomeksi tai englanniksi.
Clebsch-Gordan table and potentially helpful figures on the flip side of the paper.

1. (a) (1p) What is the definition of a cross section (in terms of experimentally measured quantities)?
   (b) (1p) What is a resonance?
   (c) (2p) What is the "electron number"? Is it conserved in nature? If not, how is the violation observed?
   (d) (2p) We know that π^0 and η are pseudoscalar mesons, i.e. $J^{PC} = 0^{-+}$ particles, and that the photon is a vector, i.e. 1^{--}. Out of the following 4 reactions, which 2 are forbidden due to C or P conservation in the strong and electromagnetic interactions?
      i. $\eta \to 2\pi^0$
      ii. $\eta \to 3\pi^0$
      iii. $\eta \to 2\gamma$
      iv. $\eta \to 3\gamma$

   Reminder: the parity of a state with particles a and b is $P = P_a P_b (-1)^L$.

2. The HERA accelerator at DESY in Germany made electron-proton collision experiments with energies $E_e = 30$ GeV and $E_p = 920$ GeV. You can assume that the proton and electron are massless. Consider an elastic interaction: $e + p \to e + p$. If the scattering angle of the outgoing electron with respect to the direction of the incoming electon in the (laboratory) frame where the beam energies are given above is 60°; i.e. $\cos\theta = 1/2$, what is the scattering angle of the outgoing electron in the CMS frame? Draw a figure!

3. The following reactions are *not* possible, at least in the standard model, why?
   (a) (1p) $e^- + \bar{\nu}_\mu \to \nu_e + \mu^-$
   (b) (1p) $e^+ + e^- \to \gamma$

   The following reactions are possible. What interactions cause them? (If they can happen through different interactions, name the strongest/most likely one.) For the electroweak ones draw one of the Feynman diagrams by which the reaction can happen. For the strong ones draw a quark diagram; is a resonance possible?

   (c) (1p) $K^- + p \to \Sigma^- + \pi^+$
   (d) (1p) $K^+ \to \pi^0 + e^+ + \nu_e$
   (e) (1p) $\pi^0 \to 2\gamma$
   (f) (1p) $\nu_\mu + n \to \mu^- + p$

4. Consider pion-nucleon scattering at the CMS energy $\sqrt{s} = m_\Delta = 1232$MeV. Show that isospin symmetry leads to the following ratio of the cross sections:

$$\sigma(\pi^+ + p \to \pi^+ + p) : \sigma(\pi^- + p \to \pi^0 + n) : \sigma(\pi^- + p \to \pi^- + p) = 9 : 2 : 1. \qquad (1)$$

   You may use the known isospin assignments

$$-|\pi^+\rangle, |\pi^0\rangle, |\pi^-\rangle = |1,1\rangle, |1,0\rangle, |1,-1\rangle \qquad (2)$$

$$|p\rangle, |n\rangle = \left|\frac{1}{2}, \frac{1}{2}\right\rangle, \left|\frac{1}{2}, -\frac{1}{2}\right\rangle \qquad (3)$$

$$|\Delta^{++}\rangle, |\Delta^+\rangle, |\Delta^0\rangle, |\Delta^-\rangle = \left|\frac{3}{2}, \frac{3}{2}\right\rangle, \left|\frac{3}{2}, \frac{1}{2}\right\rangle, \left|\frac{3}{2}, -\frac{1}{2}\right\rangle, \left|\frac{3}{2}, -\frac{3}{2}\right\rangle, \qquad (4)$$

   and the information on the cross section at the resonance peak $\sqrt{s} \approx m_R$

$$\sigma_{ab\to cd} = \frac{\pi}{(q_i^{\mathrm{TRF}})^2} \frac{\Gamma_{R\to ab}\Gamma_{R\to cd}}{(\sqrt{s} - m_R)^2 + \Gamma^2/4}, \qquad (5)$$

   where $\Gamma_{R\to ab}$ is the decay width for the resonance decay $R \to ab$.

36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONIC AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2			
m_1	m_2	Coefficients		
.	.			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M$$

1/2×1/2

m_1	m_2	1, +1
+1/2	+1/2	1

m_1	m_2	1, 0	0, 0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	1, −1
−1/2	−1/2	1

1×1/2

m_1	m_2	3/2, +3/2
+1	+1/2	1

m_1	m_2	3/2, +1/2	1/2, +1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	3/2, −1/2	1/2, −1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	3/2, −3/2
−1	−1/2	1

2×1

m_1	m_2	3, +3
+2	+1	1

m_1	m_2	3, +2	2, +2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	3, +1	2, +1	1, +1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

m_1	m_2	3, 0	2, 0	1, 0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	3, −1	2, −1	1, −1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	3, −2	2, −2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	3, −3
−2	−1	1

1×1

m_1	m_2	2, +2
+1	+1	1

m_1	m_2	2, +1	1, +1
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	2, 0	1, 0	0, 0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	2, −1	1, −1
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	2, −2
−1	−1	1

3/2×1

m_1	m_2	5/2, +5/2
+3/2	+1	1

m_1	m_2	5/2, +3/2	3/2, +3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

m_1	m_2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

m_1	m_2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

m_1	m_2	5/2, −3/2	3/2, −3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

m_1	m_2	5/2, −5/2
−3/2	−1	1

2×1/2

m_1	m_2	5/2, +5/2
+2	+1/2	1

m_1	m_2	5/2, +3/2	3/2, +3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

m_1	m_2	5/2, +1/2	3/2, +1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	5/2, −1/2	3/2, −1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	5/2, −3/2	3/2, −3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

m_1	m_2	5/2, −5/2
−2	−1/2	1

3/2×1/2

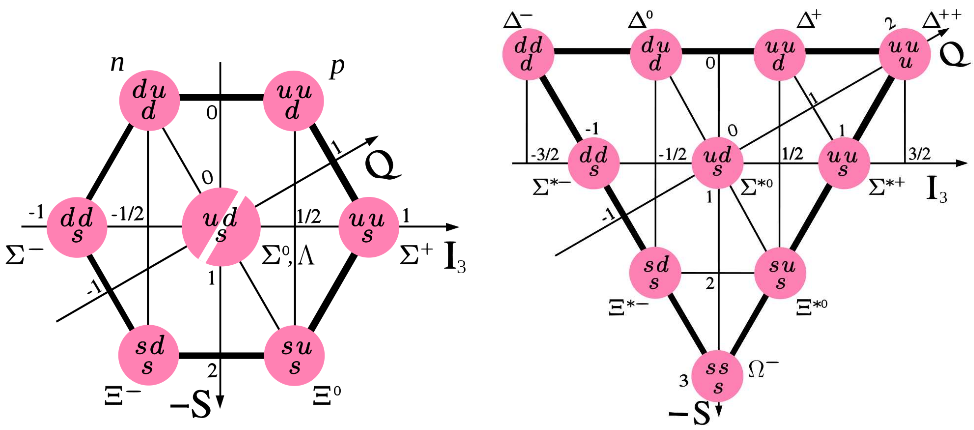
m_1	m_2	2, +2
+3/2	+1/2	1

m_1	m_2	2, +1	1, +1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

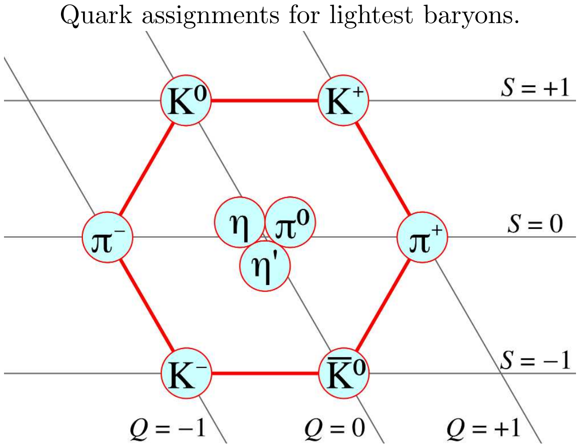
m_1	m_2	2, 0	1, 0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	2, −1	1, −1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	2, −2
−3/2	−1/2	1



Quark assignments for lightest baryons.



Lightest meson nonet (pseudoscalar mesons consisting of one u, d or s quark and one $\bar{u}, \bar{d}$ or $\bar{s}$ antiquark.) Reminder: strange quark has $S = -1$. From S and the electric charge Q of the meson you can reconstruct the quark content.