FYSH300 Particle physics
2. half-course exam (2. välikoe) 13.12.2013: 4 problems, **4 hours**.

1. a) What is meant by deep inelastic electron-proton scattering, and what is the main purpose of such collision measurements? (1p)

b) Draw a detailed parton-model graph of the Standard Model (SM) Higgs production&decay process, where the Higgs is produced in $p+p$ collisions through heavy-vector-boson fusion with tagged jets (one in the forward direction and one in the backward direction), and where the Higgs then decays into a muon-antimuon pair and 2 jets. All the vertices appearing in your graph must be basic SM vertices – consult the attachments if needed. Identify the colliding protons and partons, beam jets, tagged jets, decay-jets and all other particles in your graph. (1p)

c) Let's look at the SM Higgs measurements shown in the figures below.
Left: The invariant mass distribution of four leptons $(l^+l^-l^+l^-)$
Right: The invariant mass distribution of $e^{\pm}\mu^{\mp}$.
Explain why in the four-lepton channel there is a peak at $m_{llll} \approx 125$ GeV, while in the $e\mu$ channel such a peak is not seen at $m_{ll} \approx 125$ GeV. (1p)

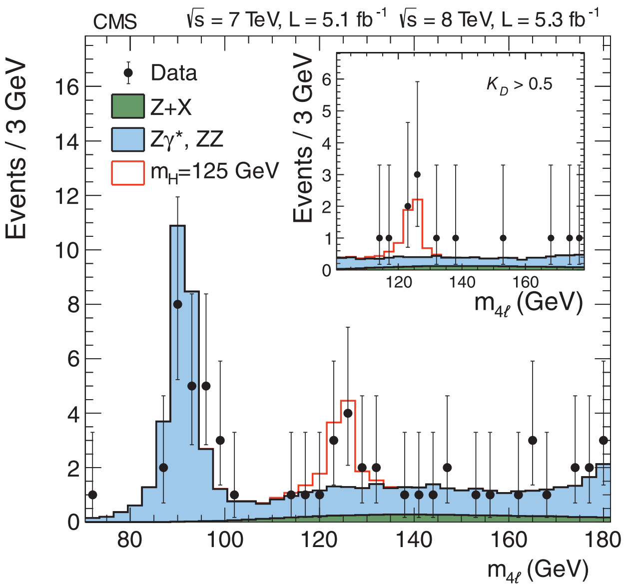


Fig. 4. Distribution of the four-lepton invariant mass for the ZZ → 4ℓ analysis. The points represent the data, the filled histograms represent the background, and the open histogram shows the signal expectation for a Higgs boson of mass $m_H = 125$ GeV, added to the background expectation. The inset shows the $m_{4\ell}$ distribution after selection of events with $K_D > 0.5$, as described in the text.

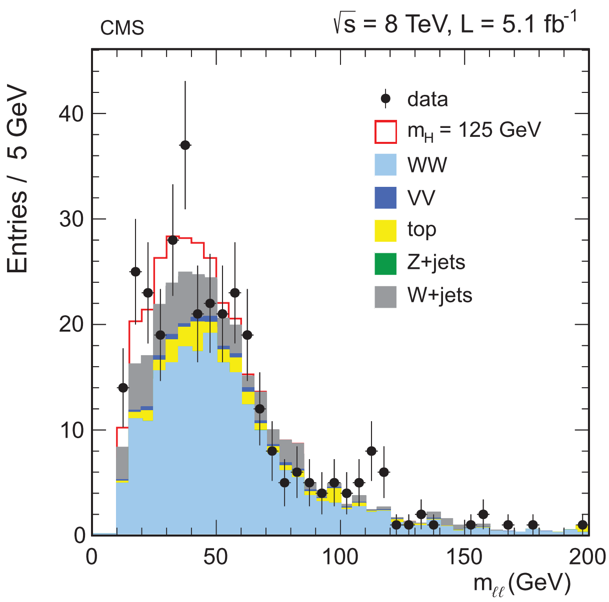


Fig. 7. Distribution of $m_{\ell\ell}$ for the zero-jet eμ category in the H → WW search at 8 TeV. The signal expected from a Higgs boson with a mass $m_H = 125$ GeV is shown added to the background.

d) Sketch a cross-section figure of the LHC's CMS detector, where you put the beam pipe (BP), muon detector (MD), electromagnetic calorimeter (ECAL), tracking chamber (TC), and hadron calorimeter (HCAL), at their correct places. (1p)

e) The absolute values of the elements of the CKM-matrix have been measured to be as follows (ignoring the experimental error bars here):

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97427 & 0.22534 & 0.00351 \\ 0.22520 & 0.97344 & 0.0412 \\ 0.00867 & 0.0404 & 0.999146 \end{pmatrix}$$

Using this, estimate the following ratio of the B^+ meson decay widths:

$$\frac{\Gamma(B^+(u\bar{b}) \to K^+(u\bar{s}) + \pi^0(u\bar{u}))}{\Gamma(B^+(u\bar{b}) \to \pi^+(u\bar{d}) + \overline{D^0}(u\bar{c}))}$$

where the quark content of the hadrons is shown in the parentheses, and where you don't have to think about any phase-space effects. Draw also the Feynman graphs for these processes. (2p)

2. Consider a theory whose Lagrange density is

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\mu^2}{2}\phi^2 - \frac{\eta}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4 + i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi + ig\bar{\psi}\gamma^5\psi\phi$$

where ϕ is a real field for a spin-0 particle which we call here ϕ, and ψ is the Dirac spinor for a spin-$\frac{1}{2}$ particle which we call here F. The γ_μ are the Dirac gamma matrices and γ^5 is given in the collection of the formulae. As usual, $\bar{\psi} \equiv \psi^\dagger\gamma^0$. The constants μ^2, η, λ, m and g are positive. Let's also assume that the potential of the theory has the minimum at $\phi = 0$, so that the particle content, masses and interactions of the theory can be directly read off from the above Lagrangian.

a) Identify the kinetic terms, mass terms and interaction terms of this theory. (1p)

b) For each interaction term, draw the basic vertex, indicate the particles participating in the vertex, and indicate also the interaction strength of each vertex. (2p)

c) Using the basic vertices of this theory [be extra careful in the b)-item above!], draw all the tree-level (=non-loop) Feynman diagrams which this theory predicts for the following scatterings, and indicate for each graph what is its dependence on the interaction strengths (3p):

i) $F + \bar{F} \to F + \bar{F}$

ii) $F + \bar{F} \to \phi + \phi$

iii) $\phi + \phi \to \phi + \phi$

3. Consider a theory whose Lagrange density is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2,$$

where ϕ is a complex scalar field which depends on the coordinate 4-vector x and which describes a charged spin-0 particle. The gauge field is A_μ and the field strength tensor is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This theory is invariant in local U(1) gauge (phase) transformations,

$$\phi \xrightarrow{U(x)} U(x)\phi, \quad \text{where} \quad U(x) = e^{i\alpha(x)}$$

where $\alpha(x)$ is real. The covariant derivative, $D_\mu = \partial_\mu - ieA_\mu$, is required to transform in these gauge transformations as

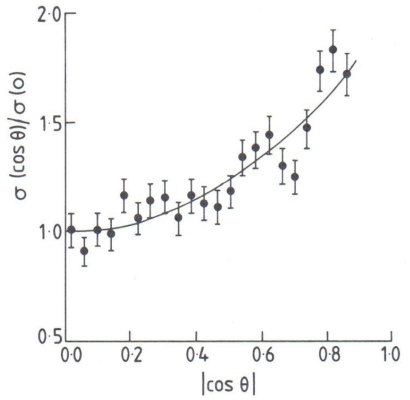
$$D_\mu\phi \xrightarrow{U(x)} U(x)D_\mu\phi.$$

a) Derive the transformation law for the gauge field A_μ in these gauge transformations. (2p)

b) To test your understanding of the 2013 Physics Nobel prize(!): Let's assume that $\lambda > 0$ but $\mu^2 < 0$, so that the Higgs mechanism is needed to find out the physical fields and masses of the theory. <u>Perform the Higgs mechanism for this theory in detail.</u> In the end, identify the physical particle content of the theory, the Higgs particle and the masses of the particles — obtaining the masses^2 up to factors 2 is just fine, you don't need to consider Euler-Lagrange equations here. (4p) *Hint:* Start by writing $\phi(x) = U(x)^{-1}\tilde{\phi}(x)$ where $\tilde{\phi}(x)$ is a real field which accounts for the oscillations around the potential's minimum at $|\phi|^2_{\text{min}} = \frac{v^2}{2} \equiv \frac{-\mu^2}{2\lambda}$.

4. The figure below describes the measurement of the angular distribution (in the CMS frame) of muons produced in the unpolarized scattering

$$e^+ + e^- \to \mu^+ + \mu^-$$



Starting from the Feynman rules, compute the differential cross section $d\sigma/d\cos\theta^*$ of this process, in the lowest order in the electromagnetic interaction. Express the final result in terms of the scattering angle θ^* in the CMS-frame, fine-structure constant $\alpha = \frac{e^2}{4\pi}$ and the CMS energy $\sqrt{s}$, assuming that $\sqrt{s}$ is in the range 15—40 GeV. After this, form the quantity

$$\frac{\frac{d\sigma}{d\cos\theta^*}}{\frac{d\sigma}{d\cos\theta^*}\big|_{\cos\theta^*=0}},$$

which is shown in the figure and compare your result with the figure. In the end, explain briefly what one can learn from measuring such an angular distribution.

Instructions: Let's consider here only the high-energy limit, i.e. you can set the particle masses in the initial and final states to zero. Please use the following notation: p_a for the 4-momentum of the positron, p_b for the 4-momentum of the electron, p_c for the antimuon and p_d for the muon. Mark all the intermediate steps you perform in your answer sheet – and not on a scrap paper. Make use of the attached table (Field C.1) of the Feynman rules and the collection of the formulae.

Collection of formulae

$$g_{\mu\nu} = g^{\mu\nu} \hat{=} diag(1,-1,-1,-1)$$

$$A^\mu B_\mu = A^0 B^0 - \mathbf{A}\cdot\mathbf{B}$$

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\mathbf{1}_4$$

$$\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0$$

$$\gamma_\mu\gamma^\mu = 4\mathbf{1}_4$$

$$\gamma_\mu \not{a}\, \gamma^\mu = -2\not{a}, \text{ where } \not{a} \equiv \gamma_\mu a^\mu$$

$$\gamma_\mu \not{a} \not{b}\, \gamma^\mu = 4a\cdot b$$

$$\gamma_\mu \not{a} \not{b} \not{c}\, \gamma^\mu = -2\not{c} \not{b} \not{a}$$

$$\gamma^{5\dagger} = \gamma^5 \text{ , where } \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$(\gamma^5)^2 = \mathbf{1}_4$$

$$\{\gamma^5, \gamma^\mu\} = 0$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})$$

$$\mathrm{Tr}(\gamma^5) = 0$$

$$\mathrm{Tr}(\gamma^{\mu_1}\gamma^{\mu_2}\ldots\gamma^{\mu_{2n+1}}) = 0$$

Projection operators for Dirac spinors:

$$\sum_{s=1,2} u^{(s)}(p)\overline{u}^{(s)}(p) = \not{p} + m \qquad \sum_{s=1,2} v^{(s)}(p)\overline{v}^{(s)}(p) = \not{p} - m,$$

where $\overline{u} \equiv u^\dagger\gamma^0$ and $\overline{v} \equiv v^\dagger\gamma^0$

Dirac equations: $(\not{p} - m)u(p) = 0 \qquad (\not{p} + m)v(p) = 0$

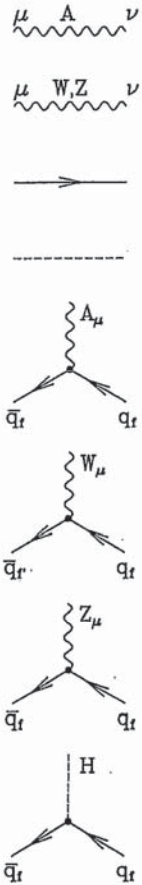
Cross section $ab \to cd$ (when $m_{a,b} = m_{c,d}$):

$$\frac{d\sigma}{d\Omega^*} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s}$$

Spherical coordinates:

$$\int d\Omega = \int_0^{2\pi} d\phi \int_{-1}^{1} d(\cos\theta)$$

⇒ <u>Feynmanin säännöt</u> (ks. esim. Ellis, Stirling, Webber, QCD and Collider Physics)



$$[-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}]\frac{i}{q^2}$$

$$[-g^{\mu\nu}+\frac{q^\mu q^\nu}{M^2}]\frac{i}{(q^2-M^2)}$$

h nom. massa

$$\frac{i}{\not{q}-m}$$

$$\frac{i}{(q^2-M_H^2)}$$

$$-ieQ_f\gamma_\mu$$

$= \gamma^5$ $\quad T^+=\begin{pmatrix}0&1\\0&0\end{pmatrix}$

$$\frac{-ig_W}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)(T^+)_{f'f} \qquad \underbrace{g_W}_{=g}=\frac{e}{\sin\theta_W}$$

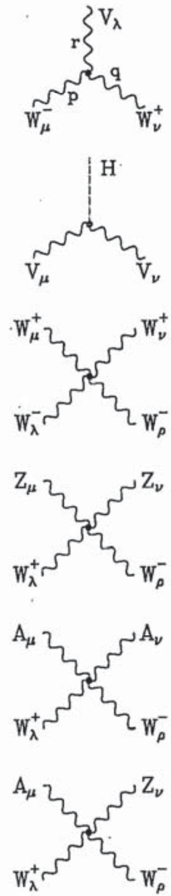
$$\frac{-ig_W}{2\cos\theta_W}\gamma^\mu(V_f-A_f\gamma_5)$$

$(V_f=T_f^3-2Q_f\sin^2\theta_W,\quad A_f=T_f^3)$

	Q_f	V_f	A_f
$q_{ylä}$	$\frac{2}{3}$	$+\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$	$+\frac{1}{2}$
q_{ala}	$-\frac{1}{3}$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}$
ν_ℓ	0	$+\frac{1}{2}$	$+\frac{1}{2}$
ℓ_ℓ	-1	$-\frac{1}{2}+2\sin^2\theta_W$	$-\frac{1}{2}$

$$\frac{-ig_W m_f}{2M_W}$$

Fig. 8.5. Propagators and Feynman rules for fermion interactions.



$$+ig_V\,[(p-q)_\lambda g_{\mu\nu}+(q-r)_\mu g_{\nu\lambda}+(r-p)_\nu g_{\lambda\mu}]$$

(all momenta incoming, $g_A=e,\ g_Z=g_W\cos\theta_W$)

$g_W = e/\sin\theta_W$

$$+ig_{VH}M_W g_{\mu\nu}$$

$(g_{WH}=g_W,\ g_{ZH}=g_W/\cos^2\theta_W)$

$$+ig_W^2[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$

$$-ig_W^2\cos^2\theta_W[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$

$$-ie^2[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$

$$-ieg_W\cos\theta_W[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$

Fig. 8.2. Feynman rules for boson interactions.

Field: Applications of perturbative QCD

C.1 Feynman Rules – General Discussion

External Lines:
For particles in the initial or final state one writes the following factor:

(a) Spin Zero Boson 1

QED → fotoni (b) Spin One Boson .... $\epsilon_\mu(\lambda)$
Here $\epsilon_\mu(\lambda)$ is the polarization 4-vector for a boson with helicity λ. For the case of a massless spin one boson propagating along the $\hat{z}$-axis with 4-momentum k_μ given by

$$k_\mu = \begin{pmatrix} k_0 \\ 0 \\ 0 \\ k_3 \end{pmatrix} \qquad (C.1.1)$$

with $k_0 = |k_3|$, the polarization 4-vectors are given by

$$\epsilon_\mu(\lambda = \pm 1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix} , \qquad (C.1.2)$$

for helicity ± 1. The polarization 4-vectors satisfy

$$k \cdot \epsilon = 0, \qquad (C.1.3)$$

and

$$\epsilon^2 = -1. \qquad (C.1.4)$$

For a massive spin 1 boson with 4-momentum given by (C.1.1) but with $k_0^2 = k_3^2 + M^2$ one also has the longitudinal state

$$\epsilon_\mu(\lambda = 0) = \frac{1}{M} \begin{pmatrix} k_3 \\ 0 \\ 0 \\ k_0 \end{pmatrix} , \qquad (C.1.5)$$

where M is the boson mass.

QED → (c) Spin 1/2 fermion of momentum p and spin s $\equiv S_z$
in initial state .... $u(p, s)$ on the right
in final state .... $\bar{u}(p, s)$ on the left

QED → (d) Spin 1/2 antifermion of momentum p and spin s
in initial state .... $\bar{v}(p, s)$ on the left
in final state .... $v(p, s)$ on the right

Internal Lines (Propagators):
Each internal line describes a particle of momentum q and mass m. Some examples are as follows: $(\varepsilon \to 0)$

(a) Spin Zero Boson

$$\frac{i}{q^2 - m^2 + i\epsilon} \qquad (C.1.6)$$

QED → (b) Photon (Feynman Gauge)

$$\frac{-i g_{\mu\nu}}{q^2 + i\epsilon} \qquad (C.1.7)$$

(c) Spin One Boson

$$\frac{-i\left(g_{\mu\nu} - q_\mu q_\nu / m^2\right)}{q^2 - m^2 + i\epsilon} \qquad (C.1.8)$$

QED → (d) Spin 1/2 fermion

$$\frac{i(\not{q} + m)}{q^2 - m^2 + i\epsilon} \qquad (C.1.9)$$

For antifermions one uses the same propagator, treating the antifermion as a fermion of opposite 4-momentum (*i.e.*, $-q$).

Vertex Factors:
For each intersection of three (or more) lines at one point there is a vertex factor which depends on the structure of the interaction Lagrangian. Some examples are shown in Fig. C.1.

(a) Three scalar boson vertex

$$-ig \qquad (C.1.10)$$

QED → (b) $ee\gamma$ vertex

$$-ie\gamma_\mu \qquad (C.1.11)$$

Here e is the charge of the electron and the fine structure constant $\alpha = e^2/(4\pi)$.

(c) Charged spin zero boson-photon vertex

$$-iQ(p_1 + p_2)_\mu \qquad (C.1.12)$$

(d) Four point coupling for charged spin zero boson-photon

$$2iQ^2 g_{\mu\nu} \qquad (C.1.13)$$

Loops and Combinatorics:

(a) For each loop with undetermined momentum k .... $\int d^4k/(2\pi)^4$
Here the integral runs over all values of the momentum.

(b) For each closed fermion loop .... -1

(c) For each closed loop containing n identical bosons ...... $1/n!$

- 292 -