Experimental Methods in Particle Physics, FYSH456, November 29, 2013 Final Exam

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Instructions: Please write clearly. Do not just answer the questions, but document the thoughts leading to the answer. You may use any approximation in a calculation, provided you can explain why you use it and provided it does not alter the final result substantially. If you computed a result in two different ways which contradict each other, cross one out — otherwise none can be graded (to avoid guesswork).

Allowed tools: Pocket calculator, standard collections of mathematical and physical formulae.

Grading: Tot 60 points = 45 points for the final exam and 15 points for exercises sessions.

Final Grade	points (max 60)
0	<30
1	30-35
2	36-41
3	42-47
4	48-53
5	54-60

Final exam:

Table 1: 45 points over 20 questions. Minimum to pass this test is 30 points when no point from exercises.

1a	1b	1c	2a	2b	2c	2d	2e	3a	3b	3c	3d
2	2	2	2	2	2	2	3	1	2	2	2
4a	4b	4c	5a	5b	5c	5d	5e	5f	6a	6b	6c
2	2	2	2	1	2	1	1	1	2	2	3

1. **LHC accelerator**

   The LHC parameters:

   - Circumference $2\pi R_{\mathrm{LHC}} = 26,659$ m.
   - Bunch Crossing (BC) frequency f_{BC}=40 MHz.
   - The number of protons per one bunch $N_p = 10^{11}$.
   - Emittance (transverse diameter of the interaction diamond) $\sigma_{\mathrm{xy}} = 20\ \mu m$.
   - RF frequency f_{RF}=400 MHz.
   - Total inelastic pp cross section σ_{tot}=100 mb.

   (a) Derive the formula for the instantaneous Luminosity $\mathcal{L}$. Assume head-on-head beams and Gaussian beam profile

   - $\sigma_x = \sigma_y$ = variance of the Gauss beam profile = 20 μm
   - σ_s = variance of the Gauss bunch profile = 7.7 cm.

   and calculate the instantaneous luminosity $\mathcal{L}$ for 2808 BC filling scheme and the trigger rate.

   (b) Calculate the probability of pp interaction $\mu = PP(n_{\mathrm{p+p\ coll}} > 0)$, per bunch crossing.

   (c) What is the fraction of pileup events (pileup means more than one pp collision within one BC). What is the pileup rate in the triggered event sample with the same 2808 BC filling scheme?

2. **Particle kinematics**

   Investigate the rapidity density distribution of emitted protons of transverse momentum p_T=1 GeV/c and π^0 ($m_{\pi 0} = 134$ MeV/c^2) decay.

   (a) The rapidity density distribution of 1 GeV/c transverse momentum protons at mid-rapidity $dN_{\mathrm{proton}}/dy|_{y=0}$=1. What is the corresponding value of the pseudorapity density $dN_{\mathrm{proton}}/d\eta|_{\eta=0}$?

   (b) Sketch the dN_{proton}/dy and $dN_{\mathrm{proton}}/d\eta$ in the case of $\sqrt{s_{\mathrm{LHC}}}$=8 TeV proton-proton collisions.

   (c) Calculate the c.m. energy $\sqrt{s_{\mathrm{SPS}}}$ of the SPS fixed-target experiment with $E_{\mathrm{SPS,beam}}$ = 450 GeV/c. Sketch the net-baryon rapidity density $dN_{\mathrm{p-\bar{p}}}/dy$ in the case of $\sqrt{s_{\mathrm{LHC}}}$ and $\sqrt{s_{\mathrm{SPS}}}$.

   (d) Consider $\pi^0 \to 2\gamma$ decay. Let θ^* be angle between z–axis and momentum of the photon in the π^0 rest frame (see left Fig. 1). Let us denote $E^*_{\pm}$, $E_{\pm}$ photons energies in the rest and lab frame respectively and define asymmetry parameter

$$\alpha = \left| \frac{E_+ - E_-}{E_+ + E_-} \right| \tag{1}$$

Calculate decay photon energies ($E_\pm$) in the Lab frame and show that the angle θ between photon and π^0 momentum is

$$\cos\theta = \frac{\cos\theta^* + \beta}{1 + \beta\cos\theta^*} ,$$

where β is the velocity of π^0.

(e) Calculate the opening angle, $\Delta\phi$, distribution for π^0 of energy E_π decaying to the photon pairs of asymmetry α.

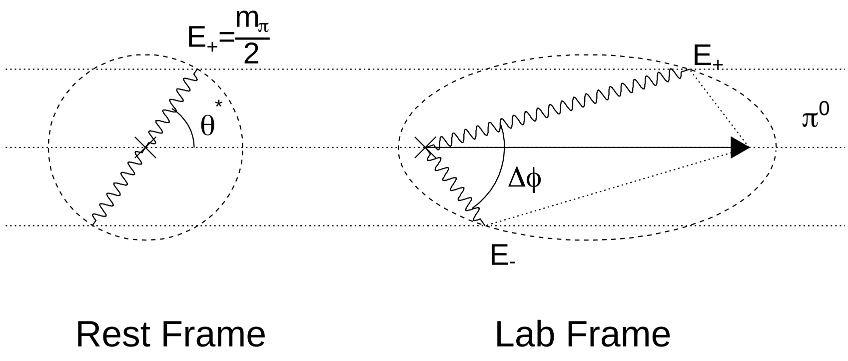

Figure 1: $\pi^0 \rightarrow 2\gamma$ in the π^0 rest frame (left) and in the Lab frame (right).

3. **p–Pb collisions at LHC**

The LHC magnets setting for p–Pb collisions allows to accelerate protons to $E_{b,p}$= 4 TeV.

(a) What is the corresponding Pb energy $E_{b,Pb}$ for the same magnet setting ($Z_{Pb} = 82$ $A_{Pb} = 208$) ?

(b) Calculate the center of mass energy per nucleon-nucleon collision $\sqrt{s_{\rm NN}}$.

(c) Calculate the rapidity of the center of mass nucleon-nucleon system $y_{\rm cm}$.

(d) Sketch pseudorapidity spectrum for bulk and high-p_T particles.

4. **Nuclear Geometry**

Let us consider symmetric AA collisions and assume, that nucleus can be approximated as a perfectly homogenous and sharp edged sphere of the radius R.

(a) Show that

$$T_A(s) = \frac{3}{2}\frac{A}{\pi R^2}\sqrt{1 - \frac{s^2}{R^2}}\;\Theta(R - s),$$

where $s \equiv |\vec{s}|$ and Θ is a step function.

(b) Show that

$$T_{AA}(\vec{0}) = \frac{9}{8}\frac{A^2}{\pi R^2} \ .$$

Hint: you may use the result for $T_A(s)$, no matter did you derive it or not.

(c) Show that at AB collisions in general, with any nuclear densities $\int d^3r \ n_A(\vec{r}) = A$ and $\int d^3r \ n_B(\vec{r}) = B$, one obtains

$$\int d^2b \ \ T_{AB}(\vec{b}) = AB.$$

5. **Nuclear modification factor** Assume the inclusive transverse momentum (p_T) distribution of the charged hadrons $h^{\pm}$ and proton-proton and Pb-Pb collisions.
   (a) Derive the form of the invariant cross section as a function of p_T.
   (b) Sketch the invariant cross section dependency on p_T measured in proton-proton collision at LHC and describe the physics which dominates various p_T ranges.
   (c) What is the shape of the p_T distribution at high-p_T in the case of ideal scale-less sigle-vector-boson exchange theory?
   (d) Why is the measured distribution different?
   (e) How is the nuclear modification factor R_{AA} defined?
   (f) Sketch the measured p_T dependency of R_{AA} at LHC and explain the and describe the physics which dominates various p_T ranges.

6. **Elliptic flow**

The azimuthal distribution of particles emerging from the non-central A–A collision can be expressed using the Fourier expansion

$$E\frac{d^3N}{dp^3} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left[1 + \sum\nolimits_{n=1}^{\infty} 2v_n(p_T)\cos n(\varphi - \psi_n)\right] \tag{5}$$

where ψ_n represents the reaction plane angle and $v_n(p_T)$ are the Fourier coefficients characterizing the particle production anisotropy in the momentum space. Consider the case when all $v_n = 0$ except v_2. Then (5) reduces to

$$\frac{dN}{d\phi} = C \cdot [1 + 2v_2 \cos 2(\varphi - \psi_2)] \tag{6}$$

where C is the normalization constant.

(a) How would you extract the v_2 coefficient from the data (follows eq. (6))?

(b) Assume you have measured the two-particle $\Delta\phi = \phi_i - \phi_j$ distribution $dN_2/d\Delta\phi$ where the only source of correlation is the $v_2 > 0$ as above. How would you write the pair double differential distribution $dN_2/d\Delta\phi$ as a function of $\Delta\phi$ angle? How would you extract the v_2 coefficient in this case?

(c) Assume the only particles you detect are the decay photons $\pi^0 \rightarrow 2\gamma$ and you know the second Fourier coefficient $v_2^{\pi 0}$ of π^0 production. What do you expect for the measured anisotropy of decay photons v_2^{γ}? Should be the v_2^{γ} value (i) larger ($v_2^{\gamma} > v_2^{\pi 0}$) or (ii) smaller ($v_2^{\gamma} < v_2^{\pi 0}$)? Explain your choice.

Supplementary

- Poisson distribution

$$P(n) = \frac{\lambda^n}{n!} e^{-\lambda}$$

- Lorentz Transformation along z-axis ($p_T = p_T^*$)

$$\begin{pmatrix} E \\ p_{||} \end{pmatrix} = \begin{pmatrix} \gamma & -\eta \\ -\eta & \gamma \end{pmatrix} \begin{pmatrix} E^* \\ p_{||}^* \end{pmatrix} \tag{8}$$

where $\eta = \gamma\beta$.

- Rapidity

$$y = \ln\left(\frac{E + p_{||}}{m_T}\right) = \frac{1}{2}\ln\left(\frac{E + p_{||}}{E - p_{||}}\right)$$

$$E = m_T \cosh y \qquad p_{||} = m_T \sinh y$$

- Pseudorapidity: In the limit $\beta \to 1$, rapidity $y \to$ pseudorapidity η

$$\eta = \frac{1}{2}\ln\left(\frac{1 + \cos\theta}{1 - \cos\theta}\right) = \frac{1}{2}\ln\left(\frac{p + p_{||}}{p - p_{||}}\right) = -\ln\tan\frac{\theta}{2}$$

$$p = p_T \cosh\eta \qquad p_L = p_T \sinh\eta$$

- Trigonometry: $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.