Ultra-Relativistic Heavy Ion Physics (FYSH551), May 31, 2013 **Final Exam**

Jan Rak and Thorsten Renk

Instructions: Please write clearly. Do not just answer the questions, but document the thoughts leading to the answer. You may use any approximation in a calculation, provided you can explain why you use it and provided it does not alter the final result substantially. If you computed a result in two different ways which contradict each other, cross one out — otherwise none can be graded (to avoid guesswork).

Allowed tools: Pocket calculator, standard collections of mathematical and physical formulae.

Table 1: 45 points over 20 questions. Minimum to pass this test is 30 points. Two additional questions 2e and 6a are for extra 3 points each.

1a	1b	1c	1d	2a	2b	2c	2d	2e	3a	3b	3d
1	2	2	2	3	2	2	2	**3**	2	3	2
4a	4b	4c	4d	5a	5b	5c	5d	5e	6a		
2	2	3	3	3	2	3	2	2	**3**		

1. **LHC accelerator**

   The LHC ring of 26,659 m circumference accelerates proton to a 4 TeV energy at bunch collision rate of 40 MHz. This means that protons are grouped in 3564 bunches per orbit with 1.2×10^{11} protons/bunch. However, in reality only 2808 bunches are used (safety reasons). The beam parameters like emittance (transverse size) and β^* parameter (longitudinal size) determine the average interaction probability, $\mu = P(n_{\text{p+p coll}} > 0)$, per one bunch crossing. For the bunch intensity $N_b = 1.2 \times 10^{11}$ p/bunch the measured interaction rate is μ=0.06 interactions/bunch.

   (a) What is the velocity of 1 tone vehicle having the same kinetic energy of all protons orbiting in LHC?

   (b) Calculate the trigger rate (assume 100% trigger efficiency) for μ=6% and the 75ns bunch crossing gap filling scheme.

   (c) What is the fraction of pileup events (pileup means more than one proton–proton collision within one bunch crossing) in the triggered event sample with the 75ns gap filling scheme?

   (d) What is the beam rapidity?

2. **Particle kinematics**

   Investigate the rapidity density distribution of emitted protons of transverse momentum p_T=1 GeV/c and π^0 ($m_{\pi 0} = 134$ MeV/c^2) decay.

   (a) The rapidity density distribution of 1 GeV/c transverse momentum protons at mid-rapidity $dN_{\text{proton}}/dy|_{y=0}$=1. What is the corresponding value of the pseudorapity density $dN_{\text{proton}}/d\eta|_{\eta=0}$?

   (b) Sketch the dN_{proton}/dy and $dN_{\text{proton}}/d\eta$ in the case of $\sqrt{s_{\text{LHC}}}$=8 TeV proton-proton collisions.

   (c) Calculate the c.m. energy $\sqrt{s_{\text{SPS}}}$ of the SPS fixed-target experiment with $E_{\text{SPS,beam}}$ = 450 GeV/c. Sketch the net-baryon rapidity density $dN_{\text{p}-\bar{\text{p}}}/dy$ in the case of $\sqrt{s_{\text{LHC}}}$ and $\sqrt{s_{\text{SPS}}}$.

   (d) Consider $\pi^0 \rightarrow 2\gamma$ decay. Let θ^* be angle between z–axis and momentum of the photon in the π^0 rest frame (see left Fig. 1). Let us denote $E^*_\pm$, $E_\pm$ photons energies in the rest and lab frame respectively and define asymmetry parameter

   $$\alpha = \left| \frac{E_+ - E_-}{E_+ + E_-} \right| \tag{1}$$

   Calculate decay photon energies ($E_\pm$) in the Lab frame and show that the angle θ between photon and π^0 momentum is

   $$\cos\theta = \frac{\cos\theta^* + \beta}{1 + \beta\cos\theta^*} \, ,$$

where β is the velocity of π^0.

(e) Calculate the opening angle, $\Delta\phi$, distribution for π^0 of energy E_π decaying to the photon pairs of asymmetry α.

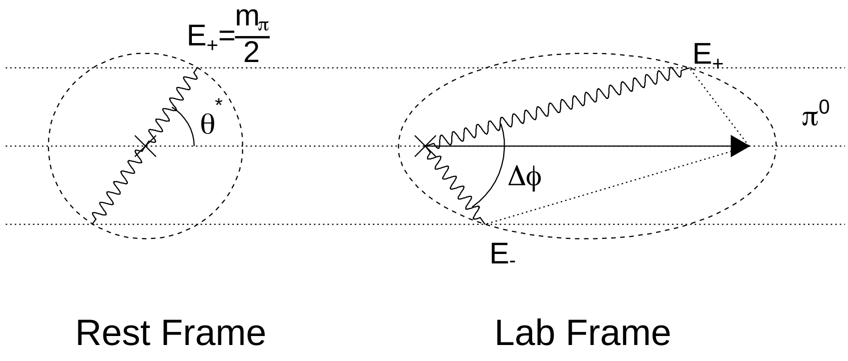

Figure 1: $\pi^0 \to 2\gamma$ in the π^0 rest frame (left) and in the Lab frame (right).

3. **Elliptic flow**

The azimuthal distribution of particles emerging from the non-central A–A collision can be expressed using the Fourier expansion

$$E\frac{d^3N}{dp^3} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left[1 + \sum_{n=1}^{\infty} 2v_n(p_T)\cos n(\varphi - \psi_n)\right] \tag{2}$$

where ψ_n represents the reaction plane angle and $v_n(p_T)$ are the Fourier coefficients characterizing the particle production anisotropy in the momentum space. Consider the case when all $v_n = 0$ except v_2. Then (2) reduces to

$$\frac{dN}{d\phi} = C \cdot [1 + 2v_2 \cos 2(\varphi - \psi_2)] \tag{3}$$

where C is the normalization constant.

(a) How would you extract the v_2 coefficient from the data (follows eq. (3))?

(b) Assume you have measured the two-particle $\Delta\phi = \phi_i - \phi_j$ distribution $dN_2/d\Delta\phi$ where the only source of correlation is the $v_2 > 0$ as above. How would you write the pair double differential distribution $dN_2/d\Delta\phi$ as a function of $\Delta\phi$ angle? How would you extract the v_2 coefficient in this case?

(c) Assume the only particles you detect are the decay photons $\pi^0 \to 2\gamma$ and you know the second Fourier coefficient $v_2^{\pi^0}$ of π^0 production. What do you expect for the measured anisotropy of decay photons v_2^γ? Should be the v_2^γ value (i) larger ($v_2^\gamma > v_2^{\pi^0}$) or (ii) smaller ($v_2^\gamma < v_2^{\pi^0}$)? Explain your choice.

4. **Hydrodynamical expansion**

Consider two systems, each at initial time $\tau_0 = 1$ fm in a cylindrical volume of radius $R_0 = 6$ fm and extending from $\eta = -1$ to $\eta = +1$ in spacetime rapidity. The single species of particles in the system is massless, the momentum k distribution isotropic and the distribution function is $f_B(E_k) = \exp[-E_k/T_0]$ where $E_k = |k|$ and $T_0 = 300$ MeV.

(a) Compute the initial rapidity distribution dN/dy of particles!

(b) In one of the systems the particles do not interact, in the other they form a thermodynamical system with an equation of state $\epsilon = 3p$. Both systems expand in longitudinal direction only with the speed of light, i.e. the volume V is proportional to proper time τ; the thermal system shows scaling flow $y = \eta$.

How does the energy density ϵ in each of the systems evolve as a function of proper time τ? Explain the difference!

(c) At time $\tau = 10$ fm, consider only particles in a small interval $d\eta$ around midrapidity $\eta = 0$. What is the rapidity distribution dN/dy of particles from this subvolume for each of the two systems now?

(d) The dilepton emission rate in the thermal system is given by

$$\frac{dN}{dtd^3\mathbf{x}dEd^3\mathbf{p}} = -\frac{1}{12\pi^4}\frac{\alpha}{M^2}\frac{1}{\exp(E/T)-1}\mathrm{Im}[\Pi^{\mathrm{ret}}{}_{\mu}{}^{\mu}(E,\mathbf{p})],$$

Assuming you can neglect $\mathrm{Im}[\Pi^{\mathrm{ret}}{}_{\mu}{}^{\mu}(E,\mathbf{p})]$ (i.e. set it to 1), determine the relative strength of dilepton emission at the ρ peak ($M = 770$ MeV) for low $P_T = 100$ MeV and high $P_T = 1$ GeV at $\tau = 1$ fm and $\tau = 10$ fm.

Give an interpretation of the result. Name at least one effect not considered in this estimate.

5. **Energy loss and nuclear suppression factor**

Consider a cylindrical volume of radius $R = 5$ fm is homogeneously filled with a free ideal gluon gas of $T = 500$ MeV. A gluon starts from the center and crosses the volume. The gluon is on an eikonal trajectory at midrapidity. Its mean free path is 0.5 fm.

(a) Estimate the scattering cross section for the gluon from the given information!

(b) By what approximate factor would the mean free path of a quark be different and why?

(c) Assume that the pQCD parton production spectrum follows a power law $dN/dp_T \sim 1/p_T^5$ and consider final state gluons at 10 GeV after the medium. Assume further that the gluon loses a given fraction z of its energy in each scattering. What is z if the observed nuclear suppression factor $R_{AA}(10 \text{ GeV}) = 0.3$?

(d) A real system is not a static cylinder. Assume $T = 500$ MeV holds at $\tau = 1$fm and the system expands in a Bjorken hydrodynamical evolution. How does this change the estimate for the average energy fraction lost per scattering?

(e) Explain the role of coherence in a more realistic picture and argue why this leads to an energy loss proportional to L^2 where L is the medium length.

(f) (difficult) Assume parton production points in the transverse plane are distributed by with a probability density $P(r) \sim \exp[-\frac{r^2}{(R/2)^2}]$ and no partons are produced outside the medium. How would you estimate the radius r_0 at which most of the observed gluons at 10 GeV are produced? (Hint: You do not need to solve the resulting expression for a number. It is helpful to consider only gluons propagating radially outward - but explain why!)

6. **Hadronic Thermodynamics**

Before Quantum Chromodynamics (QCD) was established as the theory of strong interactions, it was conjectured that there is a limiting temperature for any hot hadronic system, the so-called Hagedorn temperature. Explain the physics behind this idea, especially how there can be a 'highest possible' temperature, and explain how QCD changed the picture.

Supplementary

- eV $\approx 1.602\times10^{-19}$ J.
- Poisson distribution

$$P(n) = \frac{\lambda^n}{n!} e^{-\lambda}$$

- Lorentz Transformation

$$\begin{pmatrix} E \\ p_T \\ p_{||} \end{pmatrix} = \begin{pmatrix} \gamma & 0 & -\eta \\ 0 & 1 & 0 \\ -\eta & 0 & \gamma \end{pmatrix} \begin{pmatrix} E^* \\ p_T^* \\ p_{||}^* \end{pmatrix} \tag{5}$$

  where $\eta = \gamma\beta$.

- Rapidity

$$y = \ln\left(\frac{E + p_{||}}{m_T}\right) = \frac{1}{2}\ln\left(\frac{E + p_{||}}{E - p_{||}}\right)$$

$$E = m_T \cosh y \qquad p_{||} = m_T \sinh y$$

- Pseudorapidity: In the limit $\beta \to 1$, rapidity $y \to$ pseudorapidity η

$$\eta = \frac{1}{2}\ln\left(\frac{1+\cos\theta}{1-\cos\theta}\right) = \frac{1}{2}\ln\left(\frac{p + p_L}{p - p_L}\right) = -\ln\tan\frac{\theta}{2}$$

$$p = p_T \cosh\eta \qquad p_L = p_T \sinh\eta$$

- Trigonometry: $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.
- Integrals

$$K_n(z) = \int_0^\infty e^{-z\cosh t}\cosh(nt)dt \approx \sqrt{\frac{\pi}{2z}}e^{-z}(1+\dots)$$

$$I_0(z) = \frac{1}{2\pi}\int_0^{2\pi} e^{z\cos\phi}d\phi$$

$$\int x^2 e^{ax}dx = e^{ax}\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)$$

$$\int_0^\infty x^n e^{-ax}dx = \frac{\Gamma(n+1)}{a^{n+1}} \text{ for } a > 0, n > -1$$