FYSM300 SPRING 2012 FULL EXAM 29.6.2012

Time: 4 hrs. Every problem gives 15 points. Explain the steps in your calculations!

1. Explain briefly.
(a) Color center
(b) Seebeck effect
(c) Depletion layer
(d) Bravais lattice
(e) Show that the current carried by a partially filled electron band can be described either as an electron current or a hole current. Base your argumentation on the following result for the electron current density

$$\mathbf{j} = \frac{-e}{4\pi^3} \int_{k_{occ}} \mathbf{v}(\mathbf{k}) d\mathbf{k}$$

2. Consider a free-electron gas system in a plane (2D).
(a) Derive the relation between n (electron density) and k_F (Fermi wave length)
(b) Derive the relation between k_F and r_S (density parameter).
(c) Derive the formula for the density of electron states $g(\epsilon)$.

3. Show by using the Einstein approximation (= $3N$ harmonic modes, all vibration modes have the same frequency ω_0) that the heat capacity of lattice vibrations can be written as

$$C_v = \frac{\partial}{\partial T} \frac{3N\hbar\omega_0}{e^{\beta\hbar\omega_0} - 1} \tag{1}$$

where $\beta = 1/k_B T$. You can start from the partition function $Z = \sum_i e^{-\beta E_i}$ containing the harmonic vibrational energies or using the information that phonons are bosons. Examine the limiting behaviour of the heat capacity at high and low temperatures. At some point you may want to use the property of the geometric series:

$$s = \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r} \qquad when \quad |r| < 1$$

4. (a) Show that the trial function in the tight-binding model

$$\psi_{\mathbf{k}}(\mathbf{r}) = \sum_n C_{\mathbf{k}n} \phi(\mathbf{r} - \mathbf{r}_n) \ ; \ C_{\mathbf{k}n} = \frac{1}{\sqrt{N}} e^{i\mathbf{k}\cdot\mathbf{r}_n}$$

fulfills the Bloch theorem.
(b) Derive the dispersion relation $\varepsilon(\mathbf{k})$ in the simple cubic (SC) crystal in the tight-binding model. You can start from the band structure term

$$\varepsilon(\mathbf{k}) = -\beta \sum_n \sum_{m \neq n} e^{i\mathbf{k}\cdot(\mathbf{R}_n - \mathbf{R}_m)}.$$

Take into account only the nearest-neighbour interactions in the crystal.
(c) Find out the relation between the electron effective mass near the band minimum, parameter β and band width W. ($cos(x) \approx 1 - \frac{1}{2}x^2$ when x is small)