

Each problem gives 6 points. Time limit 4h.

1. Explain shortly
 - a) zero point energy
 - b) van der Waals interaction
 - c) point defects in crystals
 - d) n- and p-type extrinsic semiconductors
 - e) contact potential
 - f) depletion layer

2. Primitive cell of one specific lattice can be represented with basis vectors $\mathbf{a}_1 = \frac{\sqrt{3}a}{2}\hat{x} + \frac{a}{2}\hat{y}$, $\mathbf{a}_2 = \frac{-\sqrt{3}a}{2}\hat{x} + \frac{a}{2}\hat{y}$, and $\mathbf{a}_3 = c\hat{z}$, where a and c are lattice constants. Calculate the reciprocal lattice for this unknown structure and identify what are the real space and reciprocal space lattices. What are the lattice constants of the reciprocal lattice? Take the atom basis of $\mathbf{d}_1 = 0$, $\mathbf{d}_2 = \frac{a}{2\sqrt{3}}\hat{x} + \frac{a}{2}\hat{y} + \frac{c}{2}\hat{z}$ for the above mentioned primitive cell and calculate the structure factor for the cell assuming that all the atoms have the same atomic form factor i.e. $f_i = f$ for all atoms. Structure factor for the primitive cell can be calculated as

$$S_{\mathbf{K}} = \sum_i f_i \exp(-\mathbf{K} \cdot \mathbf{r}_i),$$

where $\mathbf{K} = \mathbf{k}' - \mathbf{k}$ (scattering vector) and i denotes the atom index in the primitive cell.

3. Define the equation of motion for a periodic 1D monoatomic chain with atom mass m . Assume only the nearest neighbour harmonic interactions described by a spring constant K . Solve the dispersion relation from the equation motion. What kind of phonon branches there is in the system and why? Explain and sketch the behaviour of the phonon branches at the first Brillouin zone. Sketch the vibrational modes at the center of the Brillouin zone and at the boundary of the Brillouin zone.

4. Explain the temperature dependence of electrical conductivity in:
 - a) metals (3 p)
 - b) intrinsic and extrinsic semiconductors (3 p)

5. Calculate the width W of the depletion region at 300 K in a silicon diode, when $E_g = 1.1$ eV, donor concentration in n-region $N_d = 10^{24}$ m⁻³, acceptor concentration on p-region $N_a = 10^{24}$ m⁻³ and $\epsilon_r=12$. The area of the pn-junction is 2 mm². Consider pn-junction as a parallel-plate capacitor having thickness of W and estimate the capacitance of the junction.

$$\exp(ix) = \cos(x) + i \sin(x)$$

$$a + ar + ar^2 + ar^3 + \dots = \frac{a}{1-r}$$