MATERIALS PHYSICS II (FYSM400) FALL 2010 1. EXAM 5.11.2010

Comment the steps in your derivations and calculations!
You can write the solutions either in Finnish or in English.
Points: 1: 12p, 2: 9p, 3: 9p, max=30p

1. Explain briefly (with a few lines, with perhaps schematic drawings):
(a) Plasmon
(b) Koopman's theorem
(c) Exchange hole
(d) Landau levels
(e) Screening in the electron gas
(f) Wigner crystal

2. In the Hartree-Fock approximation the total energy per electron of the homogeneous electron gas is (SI units)

$$E/N = \frac{3}{5}\epsilon_F - \frac{3e^2 k_F}{16\pi^2\varepsilon_0}.$$

Calculate the critical value of the density parameter r_s at which the electron gas completely polarizes (meaning all the electron spins are parallel) spontaneously. In that limit the energy of the completely polarized gas becomes lower than the energy of the unpolarized gas. Give the answer in Bohr units, $a_0 = 4\pi\varepsilon_0\hbar^2/me^2$. ($r_s$ is defined through $V/N = 4\pi r_s^3/3$)

3. *A simple model for plasmons in electron gas.*
(a) Consider a case where electrons are subjected to an electric field and concentration gradient (driven by a pressure gradient). The equation of motion is then

$$mn\frac{d\mathbf{v}}{dt} = -ne\mathbf{E} - \nabla p$$

where m is the electron mass, n density, $\mathbf{E}$ the field and p pressure. Fluctuations of the density about the equilibrium value n_0 is governed by the Gauss law

$$\nabla \cdot \mathbf{E} = \frac{-e(n - n_0)}{\varepsilon_0}$$

Show that for the pressure in the free-electron gas, the following holds:

$$p = 2U/3V$$

where $U = 3nE_F/5$ is the total internal energy, E_F is the Fermi energy and V volume. Use the continuity equation

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0$$

and show that the equation of motion becomes

$$\frac{\partial^2 n}{\partial^2 t} = \frac{2E_F}{3m}\nabla^2 n - \frac{n_0 e^2}{m\varepsilon_0}(n - n_0).$$

What can you say about this equation assuming a slowly spatially varying density ($\nabla^2 n = 0$)? Read out the result for the plasmon frequency ω_p.

(b) Using the result of (a), calculate the plasmon energy $\hbar\omega_p$ (in electron volts) for typical monovalent FCC metals, like Cu, Ag and Au. The lattice parameters are: Cu: 3.61 Å; Ag: 4.09 Å; Au: 4.08 Å. 1 Å$= 10^{-10}$ m, 1 eV$= 1.6\times10^{-19}$ J, m $= 9.1\times10^{-31}$ kg, $\varepsilon_0 = 8.854\times10^{-12}$ C^2 / Nm^2, $e = 1.6 \times 10^{-19}$ C, $\hbar = 1.05 \times 10^{-34}$ Js.