MATERIALS PHYSICS II (FYSM400) FALL 2012 2. EXAM 14.12.2012

Comment the steps in your derivations and calculations!
You can write the solutions either in Finnish or in English.
Points: 1: 12p, 2: 3p, 3: 6p, 4: 9p, max=30p
Time: 4 hrs

1. Explain briefly (with a few lines, with perhaps schematic drawings).
   Answer to **6 out of 9 questions only** !
   (a) magnon
   (b) Curie temperature
   (c) SQUID
   (d) fluxon (or fluxoid)
   (e) Wigner molecule
   (f) Bloch wall
   (g) Quantum phase slip in 1D superconductors
   (h) Giant magnetoresistance
   (i) Cooper pairing in high-T_C superconductors

2. Give the three Hund's rules. Apply them in the right order and draw S, L, and J as a function of the electron number in the f-shell (1 to 14 electrons).

3. (a) Find the magnetization $M = n\langle\mu\rangle$ as a function of magnetic field and temperature for a system of noninteracting spin-1 ($S = 1$) particles with magnetic moment μ and concentration n. Assume that the single particle energy is $\mathrm{E} = -\boldsymbol{\mu}\cdot\mathbf{B}$, where $\boldsymbol{\mu} = \mu\mathbf{S}/\hbar$ and $\mathbf{B} = B\hat{\mathbf{k}}$. Hint: $\langle X\rangle = \sum_i X_i\, p\,(X_i)$, where $p(X_i)$ is the probability for the system to be in state with property X_i.
   (b) Show that $M = 2n\mu^2 B/\left(3k_b T\right)$ is the high-temperature limit for the magnetization.

Turn the page!

4. Work through the *Cooper problem.* Start from a Schrödinger equation

$$-\frac{\hbar^2}{2m}(\nabla_1^2+\nabla_2^2)\phi(\mathbf{r}_1,\mathbf{r}_2)+V(\mathbf{r}_1,\mathbf{r}_2)\phi(\mathbf{r}_1,\mathbf{r}_2)=(\epsilon+2E_F)\phi(\mathbf{r_1},\mathbf{r_2}) \quad (1)$$

where ϵ is the energy of the Cooper pair (relative to the Fermi energy), Ω is the system volume and $V(\mathbf{r}_1,\mathbf{r}_2)=V(\mathbf{r}_1-\mathbf{r}_2)$ is the interaction between the electrons in the Cooper pair (with paired spins). Take

$$\phi(\mathbf{r}_1,\mathbf{r}_2)=\phi(\mathbf{r}_1-\mathbf{r}_2)=\frac{1}{\Omega}\sum_k p(\mathbf{k})e^{i\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2)}$$

and insert into (1) and show that it leads to

$$\frac{\hbar^2k^2}{m}p(\mathbf{k})+\frac{1}{\Omega}\sum_{k'>k_F}p(\mathbf{k}')v_{kk'}=(\epsilon+2E_F)p(\mathbf{k}) \quad (2)$$

where interaction matrix element $v_{kk'}=\int V(\mathbf{r})exp(-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r})\mathrm{d}\mathbf{r}$ describes the scattering of the Cooper pair from $(\mathbf{k}_\uparrow,-\mathbf{k}_\downarrow)$ to $(\mathbf{k}'_\downarrow,-\mathbf{k}'_\uparrow)$. Next, assume that $v_{kk'}=-v_0<0$ for $0<E-E_F<\hbar\omega_D$ where $\hbar\omega_D$ is small and $E=\hbar^2k^2/2m$ and show that

$$1=\frac{v_0}{\Omega}\sum_k(-\epsilon+\hbar^2k^2/m-2E_F)^{-1}. \quad (3)$$

Replace the sum in (3) by integral $\Omega^{-1}\sum_k\to(2\pi)^{-3}\int dk$ and obtain the expression

$$1=v_0\int_{E_F}^{E_F+\hbar\omega_D}\mathrm{d}E\frac{g(E)}{2E-\epsilon-2E_F}.$$

From that show that

$$1=\frac{1}{2}v_0g(E_F)\ln|(\epsilon-2\hbar\omega_D)/\epsilon| \quad (4)$$

where $g(E_F)$ is the density of states at the Fermi level for electrons of one spin orientation. Show that in the case of weak interaction ($v_0g(E_F)\ll 1$) (4) reduces to

$$\epsilon\approx-2\hbar\omega_Dexp(-2/v_0g(E_F)).$$