MATERIALS PHYSICS II (FYSM400)   FALL 2010   FULL EXAM 18.2.2011

Comment the steps in your derivations and calculations!
You can write the solutions either in Finnish or in English.
All problems give 15 points
Time: 4 hours

1. Explain briefly (with a few lines, with perhaps schematic drawings):
(a) Magnon
(b) Cooper pair
(c) Curie temperature
(d) Fluxon (or fluxoid)
(e) Exchange hole

2. The *Koopman's theorem* gives a physical meaning for the single-electron energies ϵ_i in the Hartree-Fock equation, relating them to vertical removal energies of a given electron from the system. (Vertical process means that the removal of a given electron does not disturb the wave functions of the other electrons.) Show that in this case the change of total energy upon removal of electron k is

$$\Delta E = \langle \Phi' | H | \Phi' \rangle - \langle \Phi | H | \Phi \rangle = -\epsilon_k$$

where the "prime" indicates the state of the system after electron removal. The Hartree-Fock equation is

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) \right] \phi_i(\mathbf{r}) + \frac{e^2}{4\pi\varepsilon_0} \sum_{j\neq i} \int \frac{|\phi_j(\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|} d\tau' \phi_i(\mathbf{r}) - \frac{e^2}{4\pi\varepsilon_0} \sum_{j\neq i,\ ||} \int \frac{\phi_j^*(\mathbf{r}')\phi_i(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\tau' \phi_j(\mathbf{r}) = \epsilon_i \phi_i(\mathbf{r})$$

3. *Landau Levels.* The Hamiltonian of a free electron without spin is

$$H = \frac{1}{2m}(-i\hbar\nabla + e\mathbf{A})^2.$$

The vector potential of a uniform magnetic field $B\mathbf{z}$ is $\mathbf{A} = -By\mathbf{x}$ in the Landau gauge. Search for an eigenfunction of the Schrödinger equation $H\psi = \varepsilon\psi$ in the form

$$\psi(x,y,z) = \chi(y)\exp[i(k_x x + k_z z)]$$

and show that $\chi(y)$ satisfies the equation

$$-\frac{\hbar^2}{2m}\chi''(y) + \frac{1}{2}m\omega_c^2(y-y_0)^2\chi(y) = (\varepsilon - \frac{\hbar^2 k_z^2}{2m})\chi(y),$$

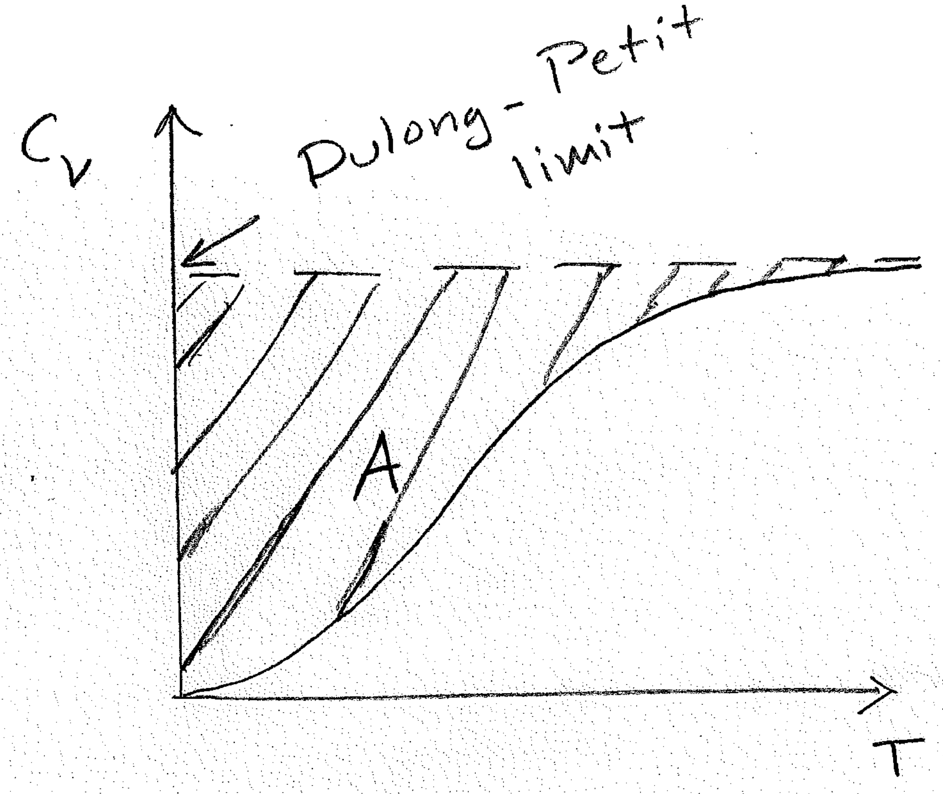
where $\omega_c = eB/m$ and $y_0 = -\hbar k_x/eB$. Argue that this equation yields the energies of the Landau levels,

$$\varepsilon_\nu = \hbar\omega_c(\nu + \frac{1}{2}) + \frac{\hbar^2 k_z^2}{2m}, \qquad \nu = 0, 1, 2, \ldots$$

4. The attached figure shows the heat capacity c_V of a system as a function of temperature. Show that the shaded area A in the figure corresponds to the quantum mechanical zero-point energy using the following equations as the starting point.

$$A = \int_0^\infty [c_V(\infty) - c_V(T)]dT$$

$$u(T) = \sum_s \int_0^\infty g(w)\varepsilon_s(T)d\omega, \quad \varepsilon_s(T) = [n_s(T) + \frac{1}{2}]\hbar\omega, \quad c_V = \frac{\partial u}{\partial T}$$



C_V
Dulong-Petit limit
A
T