8 FYSM530 Sähkönjohtavuuden kvanttimekaniikka, Quantum Transport, välikoe I, Midterm I, 1.3.2013

Solve **three problems**, you have 4 hours of time. The problems are not listed in the order of difficulty! **Handouts are allowed.** You may also solve all problems. If you do, the grade is determined from the best three.

8.1

The 2D-electron gas is quite often modelled with a so called Fang-Howard wave function:

$$\psi(\mathbf{r}, z) = 2\lambda^{3/2} z \exp(-\lambda z) \exp(i\mathbf{k}_{||} \cdot \mathbf{r})/\sqrt{A}, \tag{17}$$

where $\mathbf{r}$ is the in-plane coordinate, z the confinement direction ($z = 0$ is the location of the interface), A the area, and λ a parameter which describes the thickness of the 2D layer. Note that ψ is defined only for $z \geq 0$, for $z < 0$ it is identically zero. (a) What is the electron density n as a function of z if the 2D (uniform) density is n_s? Sketch the shape of the function. In the derivation, you may use the assumption that $T = 0$, if you like. (b) Using the Poisson equation, calculate the electrostatic potential profile $V(z)$ for $z > 0$, with the boundary conditions $V(\infty) = 0$, $V'(\infty) = 0$. The semiconductor around the 2DEG is undoped. (c) If one has a gate electrode on top of a dielectric of thickness t (i.e. a voltage V_g at $z = -t$), which is on top of the 2DEG layer, calculate the capacitance per unit area between the gate and the 2DEG. Do *not* assume that the dielectric constants are the same for the gate dielectric and the semiconductor! Explain the meaning of the two terms that you should obtain. Hint: Figure out $V(z)$ everywhere and derive an expression $V_g = f(t, \lambda)$, from which the capacitance follows.

8.2

The energy subbands for a zigzag nanotube are given by

$$E(k_x) = \pm A\sqrt{k_\nu^2 + k_x^2}, \tag{18}$$

where

$$k_\nu = \frac{2\pi}{3b}\left(\frac{3\nu}{2m} - 1\right), \tag{19}$$

and $\nu = 1, 2, 3, \ldots$ and m is an integer that gives the circumference of the nanotube as $2bm$, where $2b$ is the distance between two carbon atoms perpendicular to the nanotube axis. Negative energies are allowed, as we have simply defined $E_F = 0$. (a) Can you derive a condition for the subband quantum number ν, such that the nanotube is metallic instead of semiconducting? Sketch the the subbands $\nu = 1, 2$ if $m = 3$.

Calculate the density of states $D(E) = dN/dE$, and sketch how it looks like for $m = 3$. How is it different from the quasi-1D case where $E = \hbar^2 k_x^2/(2m^*) + E_\nu$?

8.3

The figure below is an experimental plot of the 4-wire longitudinal conductance through a 2D electron gas point-contact sample in a perpendicular magnetic field of 1.4 T (*integer* Quantum Hall regime). Explain the data using the Landauer-Büttiker formalism of Quantum Hall effect. Particularly, explain the plateaus and their *fractionally* quantized conductance values (solid horizontal lines in fig. 1 correspond to values $20, 71/2, 31/3, 11/4$.) What do you think is the meaning of the dashed lines? Sketch the behavior of the conductance, if one *increases* the magnetic field B from the value $B = 1.4$ T, when $V_g = -2$ V. Assume the transitions between plateaus in G take place at exactly half quantized values of the filling factor, e.g. at $\nu = 4.5$ etc. Give also the magnetic field values of the transitions. You may also assume that this sample does not exhibit the fractional QH effect.

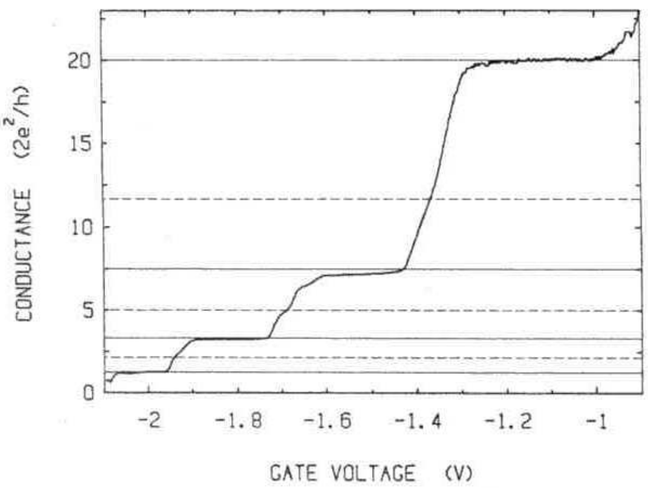

Figure 5: Longitudinal conductance G_L as a function of voltage on the point-contact gate.

8.4

Explain briefly:

(a) Why is it hard to integrate ballistic nanowires (1-D conductors) with usual metallic or semiconducting circuits (in terms of electronics not just materials science)?

(b) How does the composite fermion theory explain the fractional quantum Hall effect ?

(c) What is different about the band-structure of single-layer graphene compared with a 2D electron gas in a semiconductor heterostructure ? Can you think of at least two, maybe three reasons why device people are very interested in graphene?

(d) Explain how a semiconductor laser diode works.

(e) Give the two conditions when the Landauer -Büttiker formalism works? How can you extend the formalism if there is both a coherent an an incoherent transmission probability ? Give an expression.

(f) Why do typical heterostructure–based 2D electron systems have better high frequency operation in devices compared to bulk samples?