15 FYSM530 Sähkönjohtavuuden kvanttimekaniikka, Quantum Transport, välikoe 2, Midterm 2, 26.4.2013

Solve **three problems**, you have 4 hours of time. The problems are not listed in the order of difficulty! **Lecture notes are allowed.** Well OK, you may solve all problems. If you do, the grade is determined from the best three.

15.1

Calculate the electron tunneling probability T as a function of energy E through a 1-D wire with a single impurity modeled by a delta-function potential $V_{imp}(z) = V_0\delta(z)$ (Note units of V_0 are energy*length!). You can assume a constant effective mass everywhere. What happens if the impurity has an attractive rather than repulsive potential? Sketch the shape of the $T(E)$ function, and give its value at $E = (2m/\hbar^2)V_0^2$.

15.2

Derive the Hamiltonian of an non-symmetric ($\omega_x \neq \omega_y$) parabolic quantum dot in terms of the creation and annihilation operators $a_x, a_y, a_x^\dagger, a_y^\dagger$ satisfying $[a_i, a_i^\dagger] = 1$, and starting from the first quantized form of the Hamiltonian

$$H = \frac{\mathbf{p}^2}{2m} + V(x, y). \tag{32}$$

Give values for the energies and degeneracies if $\omega_x = 2\omega_y$ for the seven lowest energy states.

15.3

Calculate the energy required to add or remove one electron to/from the Single Electron Box, shown below. Here n is the number of excess electrons on the island, $q_g = VC_g/e$ is a normalized gate charge and ϕ is the potential of the island. Give the (discrete) values of gate voltage, where tunneling can take place at zero temperature as a function of the excess electron number. Plot also electron number as a function of V.

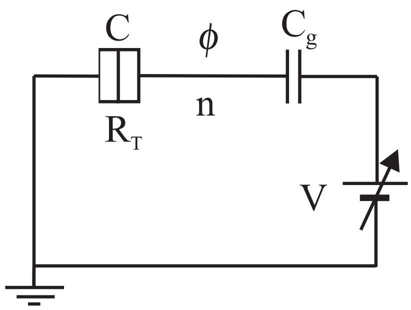

Figure 8: Schematic of the single electron box

fourth question on the back side!

15.4

Explain briefly:

(a) What is weak localization and what is its origin (why does it exist)?

(b) Why is Green's function formalism especially useful for mesoscopic transport with interactions (electron-electron, electron-phonon)?

(c) How does the Kondo effect affect transport through quantum dots and in which limit is it important?

(d) Explain the difference between coherent and incoherent transmission probablility through a system with two constrictions or junctions in series. Give an example plot for the case where the individual transmission probabilities $T_1 = T_2 = 1/2$.

(e) If you current bias a small tunnel junction by 1 pA of current, how fast will electrons tunnel?

(f) Explain what is inelastic co-tunneling through double junction structures, and why is it relevant in the Coulomb blockade regime?