# **NUCLEAR PHYSICS 1** Mid-term exam

#### **Answer 4 out of 5 questions**

1. a). Draw a diagram indicating the systematic behaviour of the nuclear binding energy per nucleon as a function of mass number A. The axes should be carefully labeled in a quantitative manner.

b) Discuss the physical basis of the various terms of the semi-empirical (Weizsäcker) mass formula.

2. a). What is meant by the term *magic numbers*? Explain briefly how they arise in single particle models of nuclei.

b). State two pieces of experimental evidence that provides evidence for shell effects in atomic nuclei.

c). In the shell model, a certain odd-parity state has total and orbital quantum numbers, *j*, and *l*, respectively. If the state has a nucleon degeneracy of 16, what are the values of *i* and *l*?

d). Using the attached shell model picture, make predictions for the spin and parity of the ground state for the following nuclei (for odd-odd cases give the possible range of spin-parities): <sup>13</sup>C (Z=6); <sup>24</sup>F (Z=9), <sup>57</sup>Ni (Z=28), <sup>102</sup>Y (Z=39)

3. a). The figure below shows the ground and first excited states of  $^{39}$ Ar (Z=18). Use the attached independent-particle model and suggest shell model configurations for the ground and first four excited states (i.e. up to the  $1/2^+$  state at 2.36 MeV).



b). Briefly explain the significance of the ratio of excitation energies of the first and second excited states of even-even nuclei. If the lowest  $2^+$  state in the deformed nucleus <sup>180</sup>Hf (Z=72) has an excitation energy of 93 keV, estimate the lowest energy of the lowest  $4^+$  state.

c). Many spherical even-even nuclei have a relatively low-lying  $I^{\pi}=3^{-}$  level. What is the significance of such a state? There is such a state in doubly-magic <sup>208</sup>Pb at an excitation energy of 2.61 MeV. What is the characteristic frequency of this state?

d). Write down the quantum mechanical expression for the energy of a rotating body as a function of spin *I*. What spin-parity values do you expect for a rotational band? The measured excitation energy of the first  $2^+$  state of  ${}^{164}$ Er is 91.4 keV. Calculate the moment of inertia. Briefly suggest why calculated excitation energies of the rotational band differ from experimental values, increasingly seen as the spin of the nucleus is increased.

- 4. The even-even nucleus <sup>212</sup>Th decays by alpha decay to the ground state of the daughter nucleus <sup>208</sup>Ra. The kinetic energy of the alpha particle emitted from <sup>212</sup>Th has been measured to be 7.802 MeV. Calculate the kinetic energy imparted to the recoil of <sup>208</sup>Ra in the alpha decay process. Given that the mass excess of <sup>208</sup>Ra is 1714 keV and the mass excess of <sup>4</sup>He is 2424.9 keV, determine the mass excess of <sup>212</sup>Th.
- a). Sketch the general shape of the electron kinetic energy spectrum for betaminus decay. Briefly indicate how the mass of the neutrino might affect this plot.
  b). Briefly explain the difference between Fermi and Gamow-Teller allowed beta decays.

c). The figure below shows a simplified version of the main beta-minus decay branches of  $^{39}$ Cl (Z=17) to  $^{39}$ Ar (Z=18).



The mass-excess of <sup>39</sup>Cl is -29800 keV/c<sup>2</sup> and that of <sup>39</sup>Ar -33242 keV/c<sup>2</sup>. Calculate the resultant  $Q_{\beta}$  value of the decay to the ground state. If the half-life of <sup>39</sup>Cl is 55.6 mins, calculate the partial half-lives of the two transitions and obtain

the log t values. Use the attached log f(Z, E0) plot and estimate the corresponding log f values for the two transitions. Finally calculate the log ft values. Using the attached table, what type of transitions have you calculated and briefly explain why the majority of the decays are to the 1517 keV transition.



**Table 3.3** Approximate values of  $\log_{10} ft_{1/2}$  for different types of  $\beta$ -decay transition.

| Type of transition | $\log_{10} ft_{1/2}$ |  |  |  |
|--------------------|----------------------|--|--|--|
| Superallowed       | ~ 3.5                |  |  |  |
| Allowed            | $5.5 \pm 1.5$        |  |  |  |
| First forbidden    | $7.5 \pm 1.5$        |  |  |  |
| Second forbidden   | $\sim 12$            |  |  |  |
| Third forbidden    | $\sim 16$            |  |  |  |
| Fourth forbidden   | $\sim 21$            |  |  |  |



## CONSTANTS

| CONSTANTS                  |                      |                                                                 |
|----------------------------|----------------------|-----------------------------------------------------------------|
| Speed of light             | с                    | 2.99792458 × 10 <sup>8</sup> m/s                                |
| Charge of electron         | e                    | 1.602189 × 10 <sup>-19</sup> C                                  |
| Boltzmann constant         | k                    | $1.38066 \times 10^{-23} \text{ J/K}$                           |
|                            | 100                  | $8.6174 \times 10^{-3} \mathrm{eV/K}$                           |
| Planck's constant          | h                    | $6.62618 \times 10^{-44} \text{ J} \cdot \text{s}$              |
|                            |                      | $4.13570 \times 10^{-13} \text{ eV} \cdot \text{s}$             |
|                            | $\hbar = h/2\pi$     | $1.054589 \times 10^{-34} \text{ J} \cdot \text{s}$             |
|                            |                      | $6.58217 \times 10^{-16} \mathrm{eV} \cdot \mathrm{s}$          |
| Gravitational constant     | G                    | $6.6726 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ |
| Avogadro's number          | NA                   | 6.022045 × 10 <sup>23</sup> mole <sup>-1</sup>                  |
| Universal gas constant     | R                    | 8.3144 J/mole · K                                               |
| Stefan-Boltzmann constant  | σ                    | $5.6703 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$          |
| Rydberg constant           | R                    | $1.0973732 \times 10^{7} \mathrm{m}^{-1}$                       |
| Hydrogen ionization energy |                      | 13.60580 eV                                                     |
| Bohr radius                | a 0                  | $.5.291771 \times 10^{-11} \text{ m}$                           |
| Bohr magneton              | μ <sub>B</sub>       | $9.27408 \times 10^{-24} \text{ J/T}$                           |
|                            |                      | 5.78838 × 10 <sup>-5</sup> eV/T                                 |
| Nuclear magneton           | μM                   | $5.05084 \times 10^{-27} \text{ J/T}$                           |
|                            |                      | $3.15245 \times 10^{-8}  \text{eV}/\text{T}$                    |
| Fine structure constant    | α                    | 1/137.0360                                                      |
|                            | hc                   | 1239.853 MeV · fm                                               |
|                            | ћc                   | 197.329 MeV · fm                                                |
|                            | $e^2/4\pi\epsilon_0$ | 1.439976 MeV · fm                                               |
|                            |                      |                                                                 |

### PARTICLE REST MASSES

|             | u                         | $MeV/c^2$ |
|-------------|---------------------------|-----------|
| Electron    | $5.485803 \times 10^{-4}$ | 0.511003  |
| Proton      | 1.00727647                | 938.280   |
| Neutron     | 1.00866501                | 939.573   |
| Deuteron    | 2.01355321                | 1875.628  |
| Alpha       | 4.00150618                | 3727.409  |
| $\pi^{\pm}$ | 0.1498300                 | 139.5669  |
| $\pi^{0}$   | 0.1448999                 | 134.9745  |
| μ           | 0.1134292                 | 105.6595  |
| CONVERSION  |                           | 2 6       |

## CONVERSION FACTORS

| $1 \text{ eV} = 1.602189 \times 10^{-19} \text{ J}$                  | 100 | $1 b = 10^{-28} m^2$                 |
|----------------------------------------------------------------------|-----|--------------------------------------|
| $1 u = 931.502 \text{ MeV}/c^2$<br>= 1.660566 × 10 <sup>-27</sup> kg |     | 1 Ci = $3.7 \times 10^{10}$ decays/s |
|                                                                      |     |                                      |