November 1, 2013

Mid-Term Exam

Answer 4 out of 5 questions

1. a) Draw a diagram indicating the systematic behavior of the nuclear binding energy per nucleon as a function of mass number A. The axes should be carefully labeled in a quantitative manner.
   b) Discuss the physical basis of the various terms of the semi-empirical (Weizsäcker) mass formula given below.

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} + \delta$$

2. The even-even nucleus ^{226}U decays by alpha decay to the ground state of the daughter nucleus ^{222}Th. The kinetic energy of the alpha particle emitted from ^{226}U has been measured to be 7.566 MeV.
   a) Calculate the kinetic energy imparted to the recoil of ^{222}Th in the alpha decay process.
   b) Given that the mass excess of ^{222}Th is 17203 keV and the mass excess of ^{4}He is 2424.9 keV, determine the mass excess of ^{226}U.
   c) The excitation energy of the first 2^+ state in ^{222}Th is 183.3 keV. What would be the kinetic energy of the alpha particle emitted in the alpha decay of ^{226}U to this state?

3. The nucleus ^{87}Y has a ground state spin and parity $I^\pi = 1/2^-$, with excited states $I^\pi = 9/2^+$ at an excitation energy of 381 keV and $I^\pi = 5/2^-$ at an excitation energy of 793 keV. The $9/2^+$ level has an experimentally measured half-life of 13 hours.
   a) Sketch the level scheme of ^{87}Y and possible transitions.
   b) What are the most likely multipolarities for the transitions?
   c) Estimate the half-lives of the excited states on the basis of the Weisskopf estimates for transition rates.
   d) If the total internal conversion coefficient for the 381 keV transition is 0.2, what is the effect on the predicted half-life of the decaying state?

4. a) State two pieces of experimental evidence which cannot be explained by the liquid-drop model of the nucleus.
   b) In a simple three-dimensional potential well, the first three energy levels have quantum numbers 1*s*, 1*p* and 1*d* in order of increasing energy. Explain what is meant by this notation. Briefly explain what additional term is required in the Woods-Saxon potential to obtain the experimentally determined magic numbers.
   c) Using the attached shell model picture, make predictions for the spin and parity of the ground state for the following nuclei (for odd-odd cases give the possible range of spin-parities):
   i. ^{13}C (Z=6), ii. ^{17}O (Z=8), iii. ^{28}Al (Z=13), iv. ^{40}K (Z=19), v. ^{59}Co (Z=27)

d) The low-lying levels in ^{49}Ca (Z=20) are: ground state, $3/2^-$; 2.02 MeV, $1/2^-$; 3.59 MeV, $5/2^-$. Interpret these states according to the shell model (note that experimentally shell model state orderings do not always follow the attached figure). Very briefly suggest why the first excited state in ^{48}Ca is at almost 4 MeV in energy.

5. a) Write down the decay energetics for β^- decay and for β^+ decay. Note that both β^+ decay and electron capture (EC) lead from the initial nucleus to the final nucleus but it is not always possible to have both. Why?
b) Briefly explain the difference between Fermi and Gamow-Teller allowed beta decays.
c) Explain what is meant by forbidden beta decay.
d) The figure below illustrates the β^- decay scheme of ^{65}Ni. The branching ratio from the parent to the $7/2^-$ state in ^{65}Cu is ~28%, to the $5/2^-$ state ~10% and to the ground state, 60%. The Q value for the beta decay is 2.137 MeV. Calculate the partial half-lives for the three branches and thus extract the log t values. Use the attached log$f$(Z,EO) plot and estimate the corresponding log f values for the three transitions. Finally calculate the log ft values. Using the attached table, what type of beta decay transitions have you calculated (superallowed, allowed, first forbidden etc)?

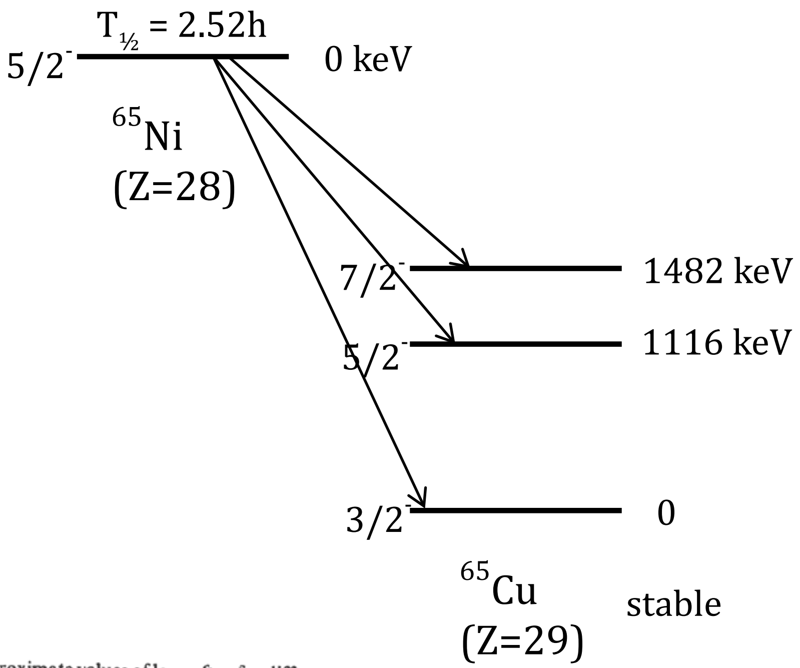
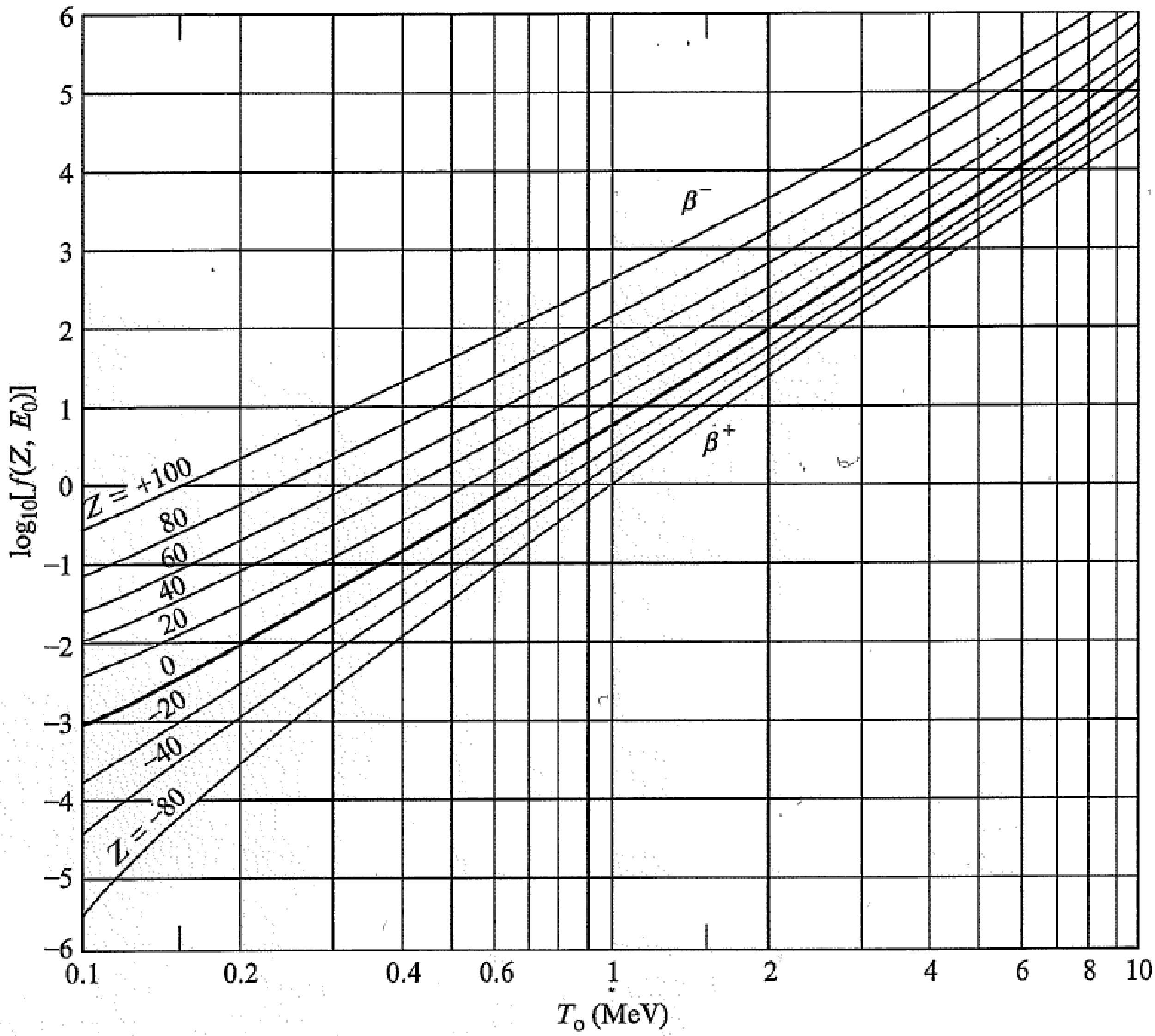


Table 3.3 Approximate values of $\log_{10} ft_{1/2}$ for different types of β-decay transition.

Type of transition	$\log_{10} ft_{1/2}$
Superallowed	~ 3.5
Allowed	5.5 ± 1.5
First forbidden	7.5 ± 1.5
Second forbidden	~ 12
Third forbidden	~ 16
Fourth forbidden	~ 21


6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
$\log_{10}[f(Z, E_0)]$
β^-
β^+
Z = +100
80
60
40
20
0
−20
−40
Z = −80
0.1
0.2
0.4
0.6
1
2
4
6
8
10
T_0 (MeV)

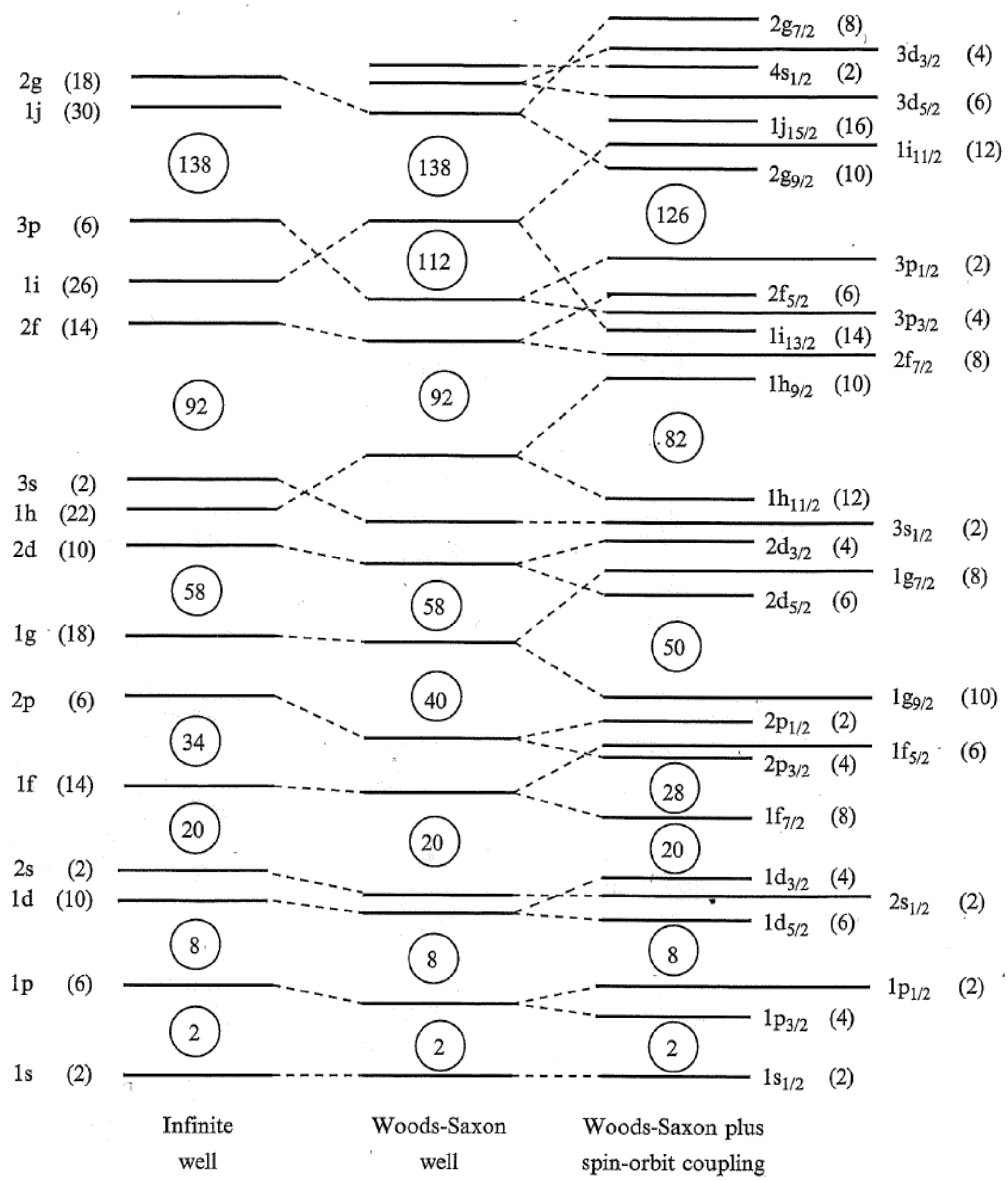


Table: Single-particle transition rate estimates (Weisskopf estimates) for electromagnetic transitions. E is the transition energy in units of MeV. The transition rate λ is given in units of s^{-1}. A is the mass number.

Electric transitions	Magnetic transitions
$\lambda(E1) = 1.0\cdot10^{14}\cdot A^{2/3}\cdot E^{3}$	$\lambda(M1) = 3.1\cdot10^{13}\cdot E^{3}$
$\lambda(E2) = 7.3\cdot10^{7}\cdot A^{4/3}\cdot E^{5}$	$\lambda(M2) = 2.2\cdot10^{7}\cdot A^{2/3}\cdot E^{5}$
$\lambda(E3) = 34\cdot A^{2}\cdot E^{7}$	$\lambda(M3) = 10\cdot A^{4/3}\cdot E^{7}$
$\lambda(E4) = 1.1\cdot10^{-5}\cdot A^{8/3}\cdot E^{9}$	$\lambda(M4) = 3.3\cdot10^{-6}\cdot A^{2}\cdot E^{9}$
$\lambda(E5) = 2.4\cdot10^{-12}\cdot A^{10/3}\cdot E^{11}$	$\lambda(M5) = 7.4\cdot10^{-13}\cdot A^{8/3}\cdot E^{11}$

CONSTANTS

Speed of light	c	2.99792458×10^{8} m/s
Charge of electron	e	1.602189×10^{-19} C
Boltzmann constant	k	1.38066×10^{-23} J/K
		8.6174×10^{-5} eV/K
Planck's constant	h	6.62618×10^{-34} J · s
		4.13570×10^{-15} eV · s
	$\hbar = h/2\pi$	1.054589×10^{-34} J · s
		6.58217×10^{-16} eV · s
Gravitational constant	G	6.6726×10^{-11} N · m^2/kg^2
Avogadro's number	N_A	6.022045×10^{23} mole^{-1}
Universal gas constant	R	8.3144 J/mole · K
Stefan-Boltzmann constant	σ	5.6703×10^{-8} W/m^2 · K^4
Rydberg constant	R_∞	1.0973732×10^{7} m^{-1}
Hydrogen ionization energy		13.60580 eV
Bohr radius	a_0	5.291771×10^{-11} m
Bohr magneton	μ_B	9.27408×10^{-24} J/T
		5.78838×10^{-5} eV/T
Nuclear magneton	μ_N	5.05084×10^{-27} J/T
		3.15245×10^{-8} eV/T
Fine structure constant	α	1/137.0360
	hc	1239.853 MeV · fm
	$\hbar c$	197.329 MeV · fm
	$e^2/4\pi\epsilon_0$	1.439976 MeV · fm

PARTICLE REST MASSES

	u	MeV/c^2
Electron	5.485803×10^{-4}	0.511003
Proton	1.00727647	938.280
Neutron	1.00866501	939.573
Deuteron	2.01355321	1875.628
Alpha	4.00150618	3727.409
$\pi^{\pm}$	0.1498300	139.5669
π^0	0.1448999	134.9745
μ	0.1134292	105.6595

CONVERSION FACTORS

1 eV = 1.602189×10^{-19} J

1 u = 931.502 MeV/c^2
    = 1.660566×10^{-27} kg

1 b = 10^{-28} m^2

1 Ci = 3.7×10^{10} decays/s