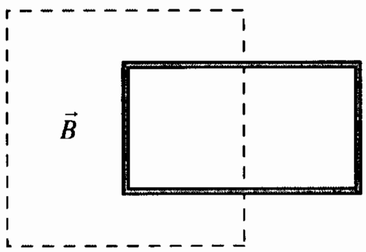
Electromagnetism (FYSP105) **27.5.2011**

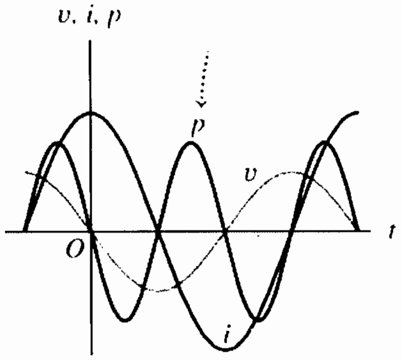
Answer all problems 1-6.

1. Answer both (a) and (b).

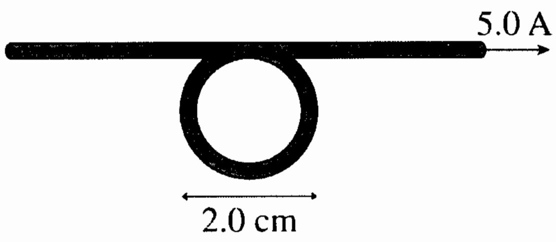
   (a) A conductor loop is placed to a magnetic field as shown in the figure. The magnetic field is confined within the area marked by the dashed line. The strength of the magnetic field starts to increase with a constant slope (T/s). Explain what happens and why. (5p)



   (b) The graph below represents the temporal behavior of voltage, current and dissipated power of a certain component in AC circuit. Which component is it? Explain your answer. (5p)



2. A long conducting wire is bent as shown. What is the magnetic field at the center of the loop? (10p)



3. Let's consider a thin conducting sheet with thickness of 0.2 mm and width of 25 mm carrying a current of 150 mA. The sheet is placed into a perpendicular magnetic field B_1 = 1.3 T (perpendicular to the the plane defined by the sheet). The induced voltage across the sheet is 100 mV. The sheet is then removed from the field and placed into another perpendicular field B_2, which induces a voltage of 59 mV.

   Calculate (a) Density of free electrons in the sheet material. (5p)

   (b) Magnitude of the magnetic field B_2. (5p)

4. Freely oscillating LC-circuit is comprised of 60 mH inductor and 250 μH capacitor. The initial charge of the capacitor is 6 μC and there is no current in the circuit at this instant. Calculate
   (a) Maximum voltage of the capacitor (2p)
   (b) Maximum current of the inductor (2p)
   (c) Maximum energy stored in the inductor (2p)
   (d) Energy stored in the inductor when the current through the inductor is half of its maximum value (2p)
   (e) Charge of the capacitor when the current through the inductor is half of its maximum value. (2p)

5. An 80 kg astronaut has gone outside his space capsule to do some repair work. Unfortunately, he forgot to lock his safety tether in place, and ha has drifted 5 m away from the capsule. Fortunately, he has a 1 kW portable laser with fresh batteries that will operate it for 1 hr. He has a 10 hr supply of oxygen. Can he make it back to the capsule? Present a detailed explanation. (10p)

6. A television channel is assigned the frequency range of 54 – 60 MHz. A series RLC tuning circuit in a TV receiver resonates in the middle of this frequency range. The circuit uses a 16 pF capacitor.

   (a) What is the value of the inductor? (4p)

   (b) In order to function properly, the current throughout the freqeuncy range must be at least 50% of the current at the resonance frequency. What is the minimum possible value of the circuit's resistance? (6p)

$$\vec{E}=\frac{\vec{F}_0}{q_0}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \qquad E=\frac{\eta}{2\varepsilon_0}$$

$$\vec{p}=q\vec{s} \qquad \vec{\tau}=\vec{p}\times\vec{E} \qquad U=-\vec{p}\cdot\vec{E}$$

$$\Phi_e=\oint_A \vec{E}\cdot d\vec{A}=\frac{Q_{\text{in}}}{\varepsilon_0}$$

$$V=\frac{U}{q_0}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

$$\Delta V=V_b-V_a=-\int_a^b \vec{E}\cdot d\vec{s} \qquad E=\frac{\Delta V}{d}$$

$$\vec{E}=-\left(\hat{i}\frac{\partial V}{\partial x}+\hat{j}\frac{\partial V}{\partial y}+\hat{k}\frac{\partial V}{\partial z}\right)=-\vec{\nabla}V$$

$$C=\frac{Q}{\Delta V}=\varepsilon_0\frac{A}{d} \qquad \varepsilon=\kappa\varepsilon_0$$

$$U_C=\frac{Q^2}{2C}=\tfrac{1}{2}C(\Delta V)^2 \qquad u=\tfrac{1}{2}\varepsilon_0 E^2$$

$$I=\frac{dQ}{dt}=nev_dA \qquad \vec{J}=ne\vec{v}_d$$

$$J=\sigma E \qquad \rho=\frac{1}{\sigma} \qquad R=\rho\frac{L}{A}$$

$$\Delta V=RI \qquad \Delta V_{\text{P}}=\mathcal{E}-Ir$$

$$P_{\text{R}}=I^2R=\frac{\Delta V_{\text{R}}^{\ 2}}{R} \qquad P_{\text{P}}=I\,\Delta V_{\text{P}}$$

$$Q(t)=C\mathcal{E}(1-e^{-t/RC}) \qquad I(t)=I_0e^{-t/\tau}$$

$$Q(t)=Q_0e^{-t/RC}=Q_0e^{-t/\tau}$$

c = 2.9979 × 10^8 m/s
e = 1.6022 × 10^{-19} C
ε_0 = 8.8542 × 10^{-12} F/m
$(4\pi\varepsilon_0)^{-1}$ = 8.9876 × 10^9 Nm²/C²
μ_0 = 4π × 10^{-7} H/m
u = 1.66054 × 10^{-27} kg = 931.49 MeV/c^2
m_e = 9.1094 × 10^{-31} kg = 0.51100 MeV/c^2
m_p = 1.6726 × 10^{-27} kg = 938.3 MeV/c^2
m_n = 1.6749 × 10^{-27} kg = 939.6 MeV/c^2
h = 6.6261 × 10^{-34} Js = 4.1357 × 10^{-15} eVs
R = 8.3145 J mol^{-1} K^{-1}
N_A = 6.0221 × 10^{23} mol^{-1}
k = 1.3807 × 10^{-23} J K^{-1}
G = 6.673 × 10^{-11} Nm²/kg²

$$\vec{B}=\frac{\mu_0}{4\pi}\frac{q\,\vec{v}\times\hat{r}}{r^2} \qquad d\vec{B}=\frac{\mu_0}{4\pi}I\frac{d\vec{s}\times\hat{r}}{r^2}$$

$$B=\frac{\mu_0}{2\pi}\frac{I}{r} \qquad B=\mu_0 nI$$

$$B=\frac{\mu_0}{2}\frac{IR^2}{(z^2+R^2)^{3/2}} \qquad B=\frac{\mu_0}{2}\frac{NI}{R}$$

$$\oint_l \vec{B}\cdot d\vec{s}=\mu_0 I_{enc} \qquad \vec{B}=\kappa_m\vec{B}_0$$

$$\vec{F}=q\vec{E}+q\vec{v}\times\vec{B} \qquad \Phi_m=\oint\vec{B}\cdot d\vec{A}=0$$

$$R=\frac{mv}{qB} \qquad f=\frac{qB}{2\pi m} \qquad \Delta V_{\text{Hall}}=\frac{IB}{tne}$$

$$d\vec{F}=I\,d\vec{s}\times\vec{B} \qquad \frac{dF}{ds}=\frac{\mu_0 I_1 I_2}{2\pi d}$$

$$\vec{\mu}=I\vec{A} \qquad \vec{\tau}=\vec{\mu}\times\vec{B} \qquad U=-\vec{\mu}\cdot\vec{B}$$

$$\mathcal{E}=-\frac{d\Phi_m}{dt}=\oint(\vec{v}\times\vec{B})\cdot d\vec{s}=\oint\vec{E}\cdot d\vec{s} \qquad \mathcal{E}=vBl$$

$$V_2/V_1=N_2/N_1 \qquad V_1I_1=V_2I_2$$

$$L=\frac{\Phi_B}{I} \qquad \mathcal{E}=-L\frac{dI}{dt}$$

$$M=\frac{\Phi_{m2}}{I_1}=\frac{\Phi_{m1}}{I_2} \qquad \mathcal{E}_2=-M\frac{dI_1}{dt}$$

$$U=\tfrac{1}{2}LI^2 \qquad u_B=B^2/2\mu_0$$

$$\omega=1/\sqrt{LC} \qquad \omega'=\sqrt{1/LC-R^2/4L^2}$$

$$I=I_0e^{-t/\tau} \qquad I=\frac{\mathcal{E}}{R}(1-e^{-t/\tau}) \qquad \tau=L/R$$

$$I_D=\varepsilon_0\frac{d\Phi_e}{dt} \qquad \oint\vec{B}\cdot d\vec{s}=\mu_0(I_C+I_D)_{enc}$$

$$E=cB \qquad c=1/\sqrt{\varepsilon_0\mu_0} \qquad n=c/v$$

$$u=\varepsilon_0E^2 \qquad \vec{S}=\frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$I=S_{av}=\frac{E_0B_0}{2\mu_0} \qquad \frac{1}{A}\frac{dp}{dt}=\frac{S}{c}=\frac{EB}{\mu_0 c}$$

$$p_{rad}=\frac{S_{av}}{c}=\frac{I}{c} \qquad p_{rad}=\frac{2S_{av}}{c}=\frac{2I}{c}$$

$$i=I\cos(\omega t) \qquad v=V\cos(\omega t+\phi)$$

$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

$$\tan\phi=\frac{\omega L-1/\omega C}{R} \qquad V=IZ$$

$$P_{av}=\tfrac{1}{2}VI\cos\phi=V_{\text{rms}}I_{\text{rms}}\cos\phi$$