FYSP112, Exam **19.12.2013**

1. Given are the following three vectors in $\mathbb{R}^3$: $\vec{A} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{B} = \hat{\imath} - \hat{k}$ and $\vec{C} = \hat{\imath} + 2\hat{\jmath}$.
   a) Calculate $\vec{A} + \vec{B} - \vec{C}$.
   b) Calculate $|\vec{A} \times \vec{B}|$.
   c) Calculate $\vec{A} \cdot \vec{B} \times \vec{C}$.

2. The position vector of a body at time t is given by

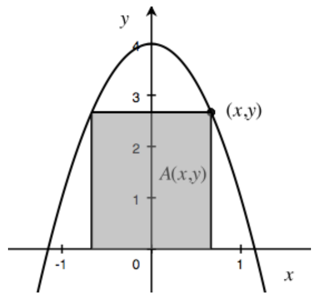
$$\vec{r}(t) = R\cos(\omega t)\hat{\imath} + R\sin(\omega t)\hat{\jmath} - \frac{1}{2}gt^2\hat{k}$$

   where R, ω and g are constants. Calculate the velocity and acceleration of the body as a function of time.

3. The function $f : \mathbb{R}^2 \to \mathbb{R}$ is defined as $f(x, y) = y \ln x + xy^2$.
   a) Calculate the partial derivatives $\frac{\partial f(x,y)}{\partial x}$ and $\frac{\partial f(x,y)}{\partial y}$.
   b) In what direction do the values of f increase most rapidly at point $(1, 2)$?
   c) Calculate the directional derivative of f at point $(1, 2)$ in direction pointing from $(1, 2)$ towards the point $(\frac{3}{2}, \frac{3}{2})$.

4. The function $f : \mathbb{R}^2 \to \mathbb{R}$ is defined as $f(x, y) = e^x \cos(x + y)$.
   a) Find the equation of the tangent plane to surface $z = f(x, y)$ at point $(0, \frac{\pi}{4})$.
   b) Use the result obtained in a) to calculate the approximative value of function f at point $(0.2, \frac{\pi}{4} - 0.1)$.

5. Use the method of Lagrange multipliers to find the maximum area of a rectangle with sides parallel to coordinate axes, and that fits entirely in the region bounded by the x axis and the parabola $y = -3x^2 + 4$ (see Figure).



6. a) Reduce the following complex number expressions in the form $x + yi$:

$$i(4 - 3i)(1 + 2i) \quad \text{and} \quad \frac{4 - 3i}{1 + 2i}.$$

   b) Express the complex number $z = 1 + \sqrt{3}i$ in a polar form $re^{i\theta}$.
   c) Calculate e^z and $\ln(z)$ (principal value) for $z = \frac{1}{\sqrt{2}}(-1 + i)$.

NOTICE 1: In all your answers, give enough details such that the principle of your solution becomes clear.
NOTICE 2: READ THE PROBLEMS CAREFULLY BEFORE ANSWERING!