FYSS311 Fundamentals of control engineering Exam

19.12.2013

1. Explain / define briefly
   (a) Transfer function
   (b) Frequency function
   (c) Open loop control
   (d) State variable
   (e) P controller
   (f) Lead compensator

2. A characteristic equation of a system is given by $s^5 + 3s^4 + 4s^3 + 6s^2 + s + K = 0$. At which values of K the system is stable?

3. A process is described by a differential equation $2\ddot{y}(t) + 5\dot{y}(t) + 2y(t) = 0$. Solve $y(t)$ as a function of time when $y(t)$ has the initial values of $y(0) = 4$ ja $\dot{y}(0) = 1$.

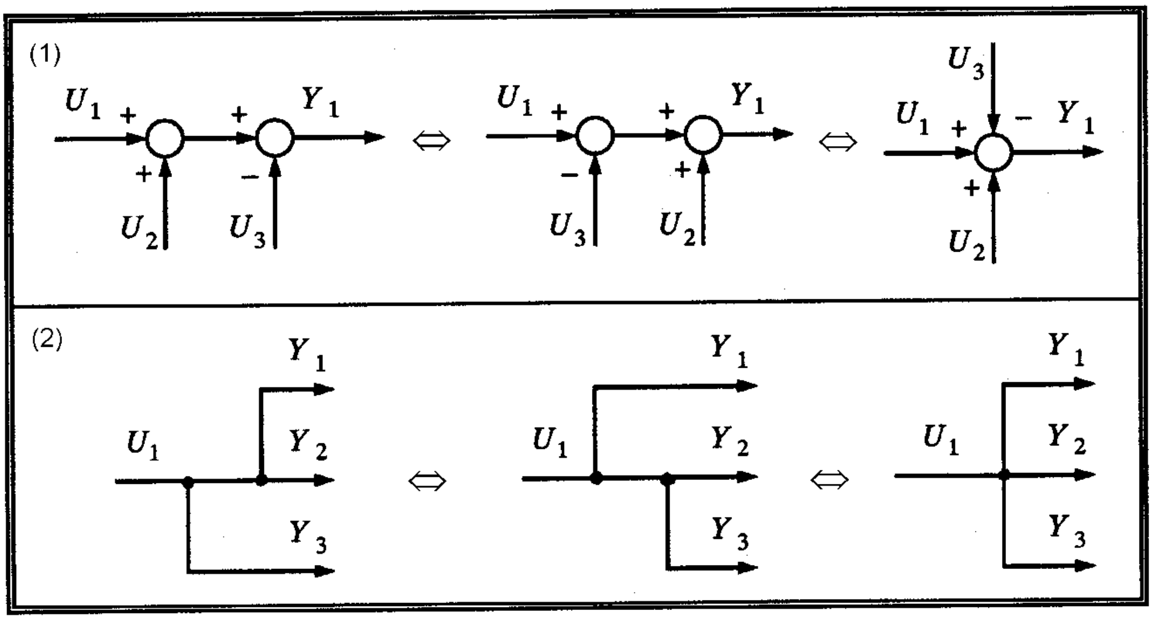
4. Consider a negative feedback system (ie. feedback = -1), that has the open loop (ie. the forward branch) transfer function given by
$$G(s) = \frac{K}{s(s+3)(s+6)}$$
Show that points $s_1 = -0.55 + i \cdot 3.0$ ja $s_2 = -1.1 + i \cdot 1.4$ belong (at a sufficient accuracy) in the root locus. Give in both cases the value of K.

5. Consider the system described in exercise 4. Give the value of K when the real parts of a complex pole pair are -0.8. What is the transfer function in this case?

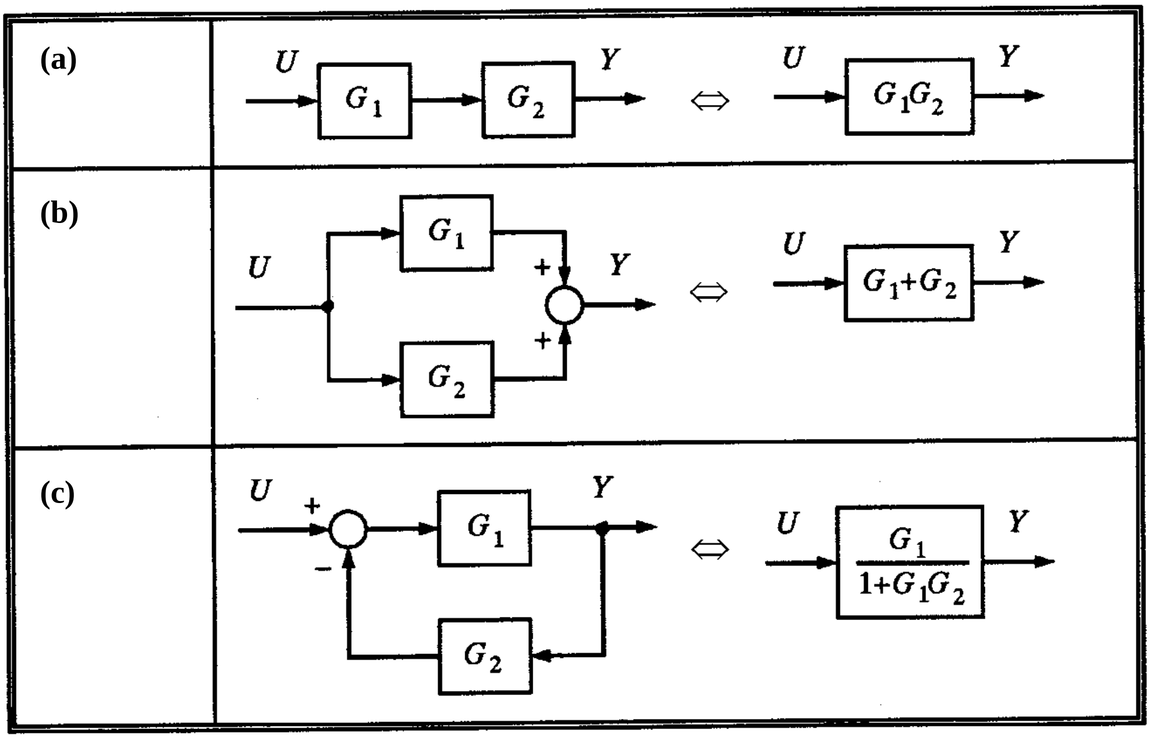
6. A sinusoidal function $u(t) = A_u \sin(\omega t)$ represents an input to a process. The process is characterized by a transfer function $G(s) = \frac{1}{s+1}$.
Determine the phase shift φ and amplitude ratio A as functions of ω,
(a) by using the response to a continuity state of the time domain and
(b) by using a frequency function.

Laplace transform	Function of time	
1	$\delta(t)$	M1
$\frac{1}{s}$	1	M2
$\frac{1}{s^2}$	t	M3
$\frac{1}{s^{n+1}}$	$\frac{t^n}{n!}$	M4
$\frac{1}{s+a}$	e^{-at}	M5
$\frac{1}{(s+a)^2}$	te^{-at}	M6
$\frac{1}{(s+a)^{n+1}}$	$\frac{t^n e^{-at}}{n!}$	M7
$\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})$	M8
$\frac{1}{(s+a)(s+b)}$	$\frac{1}{a-b}(e^{-bt}-e^{-at})$	M9
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}+\frac{1}{ab(b-a)}(ae^{-bt}-be^{-at})$	M10
$\frac{a}{s^2+a^2}$	$\sin(at)$	M11
$\frac{s}{s^2+a^2}$	$\cos(at)$	M12
$\frac{a}{(s+b)^2+a^2}$	$e^{-bt}\sin(at)$	M13
$\frac{s+b}{(s+b)^2+a^2}$	$e^{-bt}\cos(at)$	M14
$\frac{s+a}{s+b}$	$\delta(t)+(a-b)e^{-bt}$	M15

Summing points and Branch points:



(a) Cascaded system, (b) parallel system and (c) feedback system:



$$
\begin{array}{l|lllll}
n: & a_0 & a_2 & a_4 & a_6 & \dots \\
n-1: & a_1 & a_3 & a_5 & a_7 & \dots \\
n-2: & b_1 & b_2 & b_3 & b_4 & \dots \\
n-3: & c_1 & c_2 & c_3 & c_4 & \dots \\
n-4: & d_1 & d_2 & d_3 & d_4 & \dots \\
\vdots & & & & & \\
2: & e_1 & e_2 & & & \\
1: & f_1 & & & & \\
0: & g_1 & & & &
\end{array}
$$

$$
\begin{array}{lll}
b_1 = \dfrac{a_1 a_2 - a_0 a_3}{a_1} & c_1 = \dfrac{b_1 a_3 - a_1 b_2}{b_1} & d_1 = \dfrac{c_1 b_2 - b_1 c_2}{c_1} \\
b_2 = \dfrac{a_1 a_4 - a_0 a_5}{a_1} & c_2 = \dfrac{b_1 a_5 - a_1 b_3}{b_1} & d_2 = \dfrac{c_1 b_3 - b_1 c_3}{c_1} \\
b_3 = \dfrac{a_1 a_6 - a_0 a_7}{a_1} & c_3 = \dfrac{b_1 a_7 - a_1 b_4}{b_1} & d_3 = \dfrac{c_1 b_4 - b_1 c_4}{c_1} \\
\vdots & \vdots & \vdots
\end{array}
$$