Mathematical Methods in Physics III (FYS T 300) 2010

The Final Exam : Part I 15 $\overline{\text{IV}}$ - 30 $\overline{\text{IV}}$; Part II 30 $\overline{\text{IV}}$.

1. (a) Explain (with one or two sentences) the concept of

a Lie group. (b) What are the ingredients that specify

a Lie algebra ?

2. (a) Find the harmonic conjugate $v(x,y)$ to $u(x,y) =$

$x^2 - y^2 - y$, so that these functions are joined

together by $u(x,y) + i v(x,y) = f(z)$. (b) Write down

Laurent's theorem. Determine the first few terms of

the Laurent series for $f(z) = z^3 \sin\left(\frac{1}{z}\right)$ about $z = 0$.

3. (a) Derive Cauchy's integral formula by making an

appeal to an integral equation. (b) Justify. Cauchy's

integral formula is a consequence of Cauchy's residue

theorem.

$\longrightarrow$

4. (a) Describe how to solve the nonlinear integral

equation $\phi(x) = f(x) + \lambda \cdot \int_0^x \phi(x-u)\,\phi(u)\,du$.   (b) Solve

(no guessing) the Hammerstein integral equation $\phi(x) =$

$\int_0^1 xu\,[\phi(u)]^3\,du$.

5. An outline of the chief points in the Hilbert-Schmidt

theory. Consider the homogeneous integral equation

$\phi(x) = \lambda \cdot \int_a^b K(x,u)\,\phi(u)\,du$ with a real symmetric kernel.

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