Mathematical Methods in Physics III (FYST 300) 2010

The Final Exam : Part I 15 IV – 30 IV ; Part II 7 V .

1. (a) Describe how to solve the nonlinear integral equation $\phi(x) = f(x) + \lambda \cdot \int_0^x \phi(x-u)\,\phi(u)\,du$. (b) Solve (no guessing) the Hammerstein integral equation $\phi(x) = \int_{-1}^{1} x^2 u^2 [\phi(u)]^2\,du$.

2. An outline of the chief points in the Hilbert – Schmidt theory. You may consider only the homogeneous integral equation $\phi(x) = \lambda \cdot \int_a^b K(x,u)\,\phi(u)\,du$ with a real symmetric kernel.

3. (a) Derive Cauchy's integral formula by making an appeal to an integral equation. (b) Justify Cauchy's integral formula is a consequence of Cauchy's residue theorem.

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4. (a) Find the harmonic conjugate $v(x,y)$ to $u(x,y) = x^3y - xy^3$, so that these functions are joined together by the analytic function $u(x,y) + i\,v(x,y) = f(z)$. (b) Write down Laurent's theorem. Obtain the Laurent series for $f(z) = z\cos\left(\frac{1}{z}\right)$ about $z = 0$.

5. (a) Explain (with one or two sentences) the concept of a Lie group. (b) What are the ingredients that specify a Lie algebra?