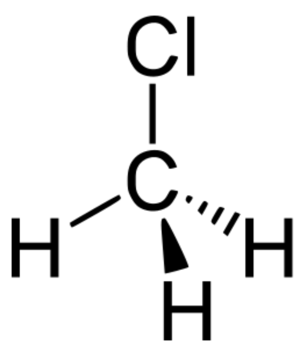
1. Consider cyclic group containing three elements, $C_3 = \langle x : x^3 = e \rangle$.
   (a) (1 p.) Consider the assignment
   $$x \mapsto \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$$
   Show that this gives a representation of C_3 on $\mathbb{C}^2$.
   (b) (3 p.) What is its character? Reduce this character in terms of irreducible characters of C_3. (Remember that all irreducible characters of cyclic group of order n are expressed in terms of n^{th} roots of 1.)
   (c) (2 p.) What are C_3-invariant subspaces in $V = \mathbb{C}^2$?

2. Consider S_3. Its character table is

	e	(12)	(123)
χ_1	1	1	1
χ_2	1	-1	1
χ_3	2	0	-1

   (a) (2 p.) S_3 has a cyclic subgroup $C_3 = \langle (123) \rangle \subset S_3$. Consider the restriction $\mathrm{Res}_{C_3}^{S_3}\chi_i$ of each irreducible character χ_i $(i = 1, 2, 3)$ of S_3 and decompose these in terms of irreducible characters of C_3.
   (b) (2 p.) Define $\mathrm{Ind}_{C_3}^{S_3}\eta$, and explain the connection between the linear operators $\mathrm{Res}_{C_3}^{S_3}$ and $\mathrm{Ind}_{C_3}^{S_3}$ given by the Frobenius reciprocity.
   (c) (2 p.) Work out $\mathrm{Ind}_{C_3}^{S_3}\eta_i$ for all irreducible characters η_i $(i = 1, 2, 3)$ of C_3, and decompose these in terms of the irreducible characters of S_3.

3. Consider CH_3Cl molecule.
   (a) (2 p.) Determine its molecular symmetry.
   (b) (3 p.) Consider its fifteen dimensional representation on the configuration space, and decompose it into irreducible representations. Remember that the trace of rotation by angle θ is $\mathrm{tr}R(\theta) = 1 + 2\cos\theta$.
   (c) (1 p.) How many vibrational eigenmodes there are?



4. Consider the following partly filled character table of group G. The conjugacy classes are denoted as C_i, $i = 1, \ldots, 5$ and the number in the second row gives the number of elements in each conjugacy class.

	C_1 1	C_2 4	C_3 5	C_4 5	C_5
χ_1					
χ_2	1	1		-1	
χ_3	1	1	-1	i	
χ_4	1	1	-1	-i	
χ_5		-1			

Answer the following question explaining clearly your steps

(a) (1 p.) Fill in the row for χ_1.

(b) (1 p.) Find $\chi_5(C_4)$.

(c) (1 p.) Find the order of G.

(d) (1 p.) Fill in the last column.

(e) (1 p.) Find $\chi_5(C_1)$.

(f) (1 p.) Complete the table.