

1. Consider cyclic group containing three elements,  $C_3 = \langle x : x^3 = e \rangle$ .

(a) (1 p.) Consider the assignment

$$x \mapsto \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$$

Show that this gives a representation of  $C_3$  on  $\mathbb{C}^2$ .

(b) (3 p.) What is its character? Reduce this character in terms of irreducible characters of  $C_3$ . (Remember that all irreducible characters of cyclic group of order  $n$  are expressed in terms of  $n^{\text{th}}$  roots of 1.)

(c) (2 p.) What are  $C_3$ -invariant subspaces in  $V = \mathbb{C}^2$ ?

2. Consider  $S_3$ . Its character table is

|          | e | (12) | (123) |
|----------|---|------|-------|
| $\chi_1$ | 1 | 1    | 1     |
| $\chi_2$ | 1 | -1   | 1     |
| $\chi_3$ | 2 | 0    | -1    |

(a) (2 p.)  $S_3$  has a cyclic subgroup  $C_3 = \langle (123) \rangle \subset S_3$ . Consider the restriction  $\text{Res}_{C_3}^{S_3} \chi_i$  of each irreducible character  $\chi_i$  ( $i = 1, 2, 3$ ) of  $S_3$  and decompose these in terms of irreducible characters of  $C_3$ .

(b) (2 p.) Define  $\text{Ind}_{C_3}^{S_3} \eta$ , and explain the connection between the linear operators  $\text{Res}_{C_3}^{S_3}$  and  $\text{Ind}_{C_3}^{S_3}$  given by the Frobenius reciprocity.

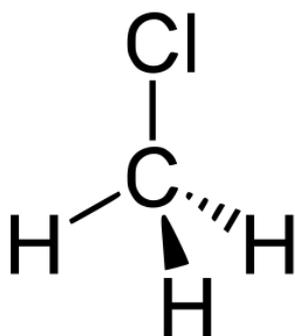
(c) (2 p.) Work out  $\text{Ind}_{C_3}^{S_3} \eta_i$  for all irreducible characters  $\eta_i$  ( $i = 1, 2, 3$ ) of  $C_3$ , and decompose these in terms of the irreducible characters of  $S_3$ .

3. Consider  $\text{CH}_3\text{Cl}$  molecule.

(a) (2 p.) Determine its molecular symmetry.

(b) (3 p.) Consider its fifteen dimensional representation on the configuration space, and decompose it into irreducible representations. Remember that the trace of rotation by angle  $\theta$  is  $\text{tr}R(\theta) = 1 + 2 \cos \theta$ .

(c) (1 p.) How many vibrational eigenmodes there are?



4. Consider the following partly filled character table of group  $G$ . The conjugacy classes are denoted as  $C_i$ ,  $i = 1, \dots, 5$  and the number in the second row gives the number of elements in each conjugacy class.

|          | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|----------|-------|-------|-------|-------|-------|
|          | 1     | 4     | 5     | 5     |       |
| $\chi_1$ |       |       |       |       |       |
| $\chi_2$ | 1     | 1     |       | -1    |       |
| $\chi_3$ | 1     | 1     | -1    | $i$   |       |
| $\chi_4$ | 1     | 1     | -1    | $-i$  |       |
| $\chi_5$ |       | -1    |       |       |       |

Answer the following question explaining clearly your steps

- (1 p.) Fill in the row for  $\chi_1$ .
- (1 p.) Find  $\chi_5(C_4)$ .
- (1 p.) Find the order of  $G$ .
- (1 p.) Fill in the last column.
- (1 p.) Find  $\chi_5(C_1)$ .
- (1 p.) Complete the table.