1. Consider a one-dimensional system [20 pt]

$$\dot{x} = x^4 + r + 1.$$

   (a) Determine and classify the fixed points of the system.

   (b) Are there any bifurcations in the system? If yes, determine the critical values of r and sketch what happens to the fixed points in the bifurcations.

2. Suppose that we have a chaotic dynamical system, so that the uncertainties in the initial positions are *(a)* $|\delta_0| = 10^{-3}$ or *(b)* $|\delta_0| = 10^{-12}$, respectively. Requiring an error tolerance of 10^{-2} in the position, how much longer in time can we predict the behavior in case (b) compared with case (a)? *[20 pt]*

3. Explain briefly the following concepts ($\lesssim$ 20 words each): *[20 pt]*

   (a) Bifurcation

   (b) Poincaré section

   (c) Liapunov exponent

   (d) Brody mixing

   (e) Logistic map

   (f) Strange attractor

   (g) Level repulsion

   (h) Gaussian unitary ensemble

   (i) Fractal

   (j) Bohigas conjecture

4. What is the essential message of the Poincaré-Bendixson theorem? Give an example (or two) of low-dimensional chaotic systems for which the theorem does not apply. *[10 pt]*

5. Explain briefly the concept of *quantum chaos* and the most important findings behind the theory (half a page at maximum). *[20 pt]*

6. Consider the billiard systems sketched below. Which of them are regular, and which are non-regular (at least partly chaotic)? Reasoning is not required. *[10 pt]*