Quantum Mechanics II Mid-exam 29.10.2010

- 1. Give a brief description of the following:
  - a. Heisenberg picture and Schrödinger picture
  - b. Accidental degeneracy
  - c. Infinitesimal generator
  - d. Irreducible representation
  - e.  $|\alpha\rangle\langle\beta|$
- 2. Consider a particle with spin quantum number s = 1. Ignore all spatial degrees of freedom and assume that the particle is subject to an external magnetic field  $B = Bu_x$ . The Hamiltonian operator of the system is  $H = gB \cdot S$ .
  - a. Construct first the matrix elements of the ladder operators  $S_+$  and  $S_-$  in the  $|1 m_s\rangle$ -basis.
  - b. Derive then the expressions for the matrices  $S_x$ ,  $S_y$  and  $S_z$  in the same basis.
  - c. If the particle is initially (at t = 0) in the state  $|11\rangle$ , find the evolved state of the particle at times t > 0.
  - d. What is the probability of finding the particle in the state  $|1-1\rangle$  at different times?
- 3. Consider two tensor operators  $T^{(k)}$  and  $U^{(k)}$ . Show that  $\sum_{q=-k}^{k} (-1)^{q} T_{q}^{(k)} U_{-q}^{(k)}$  is a

scalar operator. Show that in the case k = 1 this quantity is the same as the normal scalar product of vectors.

- 4.
- a. Show that the Pauli matrices

$$\boldsymbol{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \boldsymbol{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

obey  $\sigma_i \sigma_k = \delta_{ik} + i\varepsilon_{ikl}\sigma_l$ .

- b. Derive from this the commutation and anticommutation rules for the Pauli matrices.
- c. Show that  $(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{b}) = \vec{a} \cdot \vec{b} + i\vec{\sigma} \cdot (\vec{a} \times \vec{b})$ , where  $\vec{a}$  and  $\vec{b}$  are two vectors. Is this relation valid for any operators  $\vec{a}$  and  $\vec{b}$ ?
- 5.
- a. Show that in the Heisenberg picture an operator A varies in time according to the equation

$$\frac{dA(t)}{dt} = \frac{i}{\hbar} [H, A(t)].$$

b. The Hamiltonian of a system (an one-dimensional oscillator in en external electric field) is

$$H = \frac{p(t)^{2}}{2m} + \frac{1}{2}m\omega^{2}x(t)^{2} - eEx(t).$$

Calculate the equation of motion for the operators p(t) and x(t). Solve p(t) and x(t) in terms of p(0) and x(0).

318 35. Clebsch-Gordan coefficients

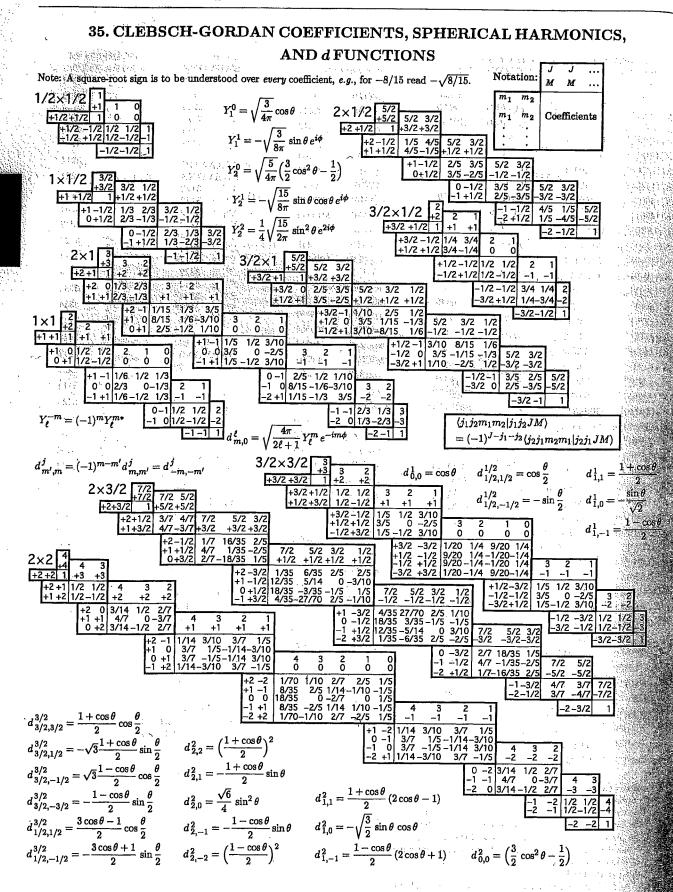


Figure 35.1: The sign convention is that of Wigner (Group Theory, Academic Press, New York, 1959), also used by Condon and Shortley (*Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (Elementary Theory of Angular Momentum, Wiley, New York, 1954), and Cohen (Tables of the Clebsch-Gordan Coefficients, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficient here have been calculated using computer programs written independently by Cohen and at LBNL.