

(You can answer in English or in Finnish.)

1. Give a brief description of the following notions:
 - a. Gauge invariance
 - b. Pauli equation
 - c. Hyperfine interactions
 - d. Zero-point energy
 - e. Stimulated emission

2. The Hamilton operator of an atom is

$$H = H_0 + \zeta \vec{L} \cdot \vec{S} + \mu_0 (\vec{L} + 2\vec{S}) \cdot \vec{B}.$$

- a. Describe briefly the origin of each term.
- b. In a strong magnetic field the atom is described mainly by

$$H_1 = H_0 + \mu_0 (\vec{L} + 2\vec{S}) \cdot \vec{B}.$$

Show that the states $|L S m_L m_S\rangle$ are eigenstates of H_1 .

- c. Consider $\xi(r)\vec{L} \cdot \vec{S}$ as a small perturbation. Evaluate the effect of this perturbation to the energies of different eigenstates of H_1 .
 - d. Sketch the energy levels of the atomic state ${}^2P_{3/2}$ under a strong magnetic field.
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3. The wave function of the electron obeys in the presence of an electromagnetic field the equation

$$[\gamma_\mu (i\partial_\mu - eA_\mu) - m]\psi = 0. \quad (1)$$

- a. Explain by which principle this equation is obtained from the Dirac equation of a free electron.
 - b. Show that the quantity $j^\mu = \bar{\psi}\gamma^\mu\psi = \psi^\dagger\gamma_0\gamma^\mu\psi = 0$.
 - c. What is the equation (the counterpart of the equation (1)) obeyed by the wave function of the positron?
 - d. How are the wave functions of the electron and the positron related to each other?
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4. In the case of a free one-electron atom, the operators H, \vec{J}^2, J_z and

$$K = \beta(1 + \vec{S} \cdot \vec{L})$$

have common eigenfunctions

$$\Psi = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}.$$

Here $\Sigma^i = \sigma^{jk}$ (cyclically) and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$.

a) Show that

$$K = \beta \left(\vec{\Sigma} \cdot \vec{j} - \frac{1}{2} \right)$$

and that the eigenvalues of K are $\kappa = \pm \left(j + \frac{1}{2} \right)$.

b) Show that

$$\vec{L}^2 \psi_A = l_A(l_A + 1) \psi_A,$$

$$\vec{L}^2 \psi_B = l_B(l_B + 1) \psi_B,$$

where $l_A = j - \frac{1}{2}$ and $l_B = j + \frac{1}{2}$ in the case $\kappa = \left(j + \frac{1}{2} \right)$ and $l_A = j + \frac{1}{2}$ and $l_B = j - \frac{1}{2}$ in the case $\kappa = - \left(j + \frac{1}{2} \right)$.

Help: $\gamma^0 \gamma^\mu \gamma^0 = (\gamma^\mu)^\dagger$.