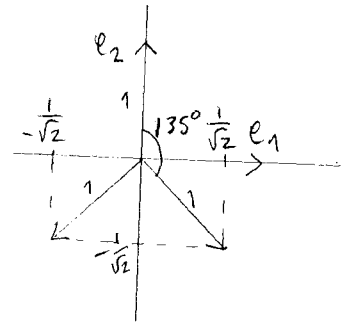


Approbetur 1A
 Harjoitusmuodi 4

1/2

1) Oletaan K kierto 225° , jolloin

$$\begin{cases} K(1,0) = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) \\ K(0,1) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) \end{cases}$$



Annetaan skalaarivertaus S on

$$S(x_1, x_2) = (2x_1, -\sqrt{2}x_2),$$

joten yhdistetty kuvaukselle $L = S \circ K$ on

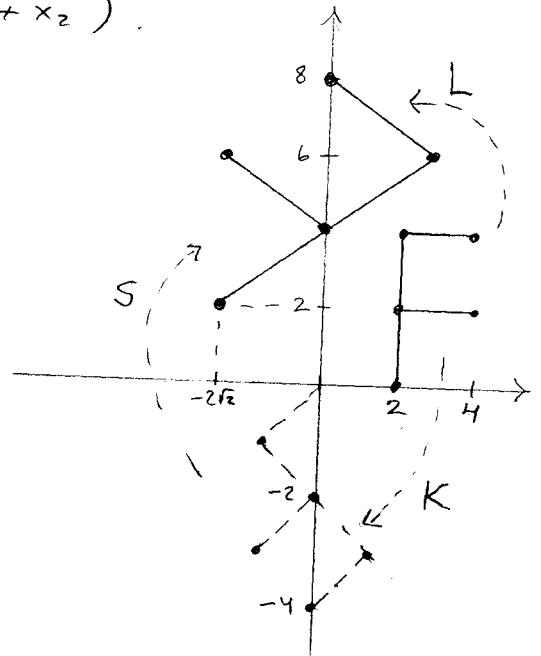
$$\begin{cases} L(1,0) = \left(-\frac{2}{\sqrt{2}}, +\frac{\sqrt{2}}{\sqrt{2}}\right) = (-\sqrt{2}, 1) \\ L(0,1) = \left(\frac{2}{\sqrt{2}}, +\frac{\sqrt{2}}{\sqrt{2}}\right) = (\sqrt{2}, 1) \end{cases}$$

Siten

$$\begin{aligned} L(x_1, x_2) &= x_1(-\sqrt{2}, 1) + x_2(\sqrt{2}, 1) \\ &= (-\sqrt{2}x_1 + \sqrt{2}x_2, x_1 + x_2). \end{aligned}$$

Kuva:

$$\begin{cases} L(2,0) = (-2\sqrt{2}, 2) \\ L(2,2) = (0, 4) \\ L(2,4) = (2\sqrt{2}, 6) \\ L(4,2) = (-2\sqrt{2}, 6) \\ L(4,4) = (0, 8) \end{cases}$$



2) Kantarektronien kuvast orat

$$\begin{cases} L(1,0,0,0,0) = (-3, 1, 2) \\ L(0,1,0,0,0) = (6, -2, -4) \\ L(0,0,1,0,0) = (-1, 2, 5) \\ L(0,0,0,1,0) = (1, 3, 8) \\ L(0,0,0,0,1) = (-7, -1, -4) \end{cases}$$

joten

$$A = \text{Mat}(L) = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

4 din Ker(L) :

2/2

$$L(x) = \vec{0} = (0, 0, 0)$$

$$\Leftrightarrow \begin{cases} -3x_1 + 6x_2 - x_3 + x_4 - 7x_5 = 0 \\ x_1 - 2x_2 + 2x_3 + 3x_4 - x_5 = 0 \\ 2x_1 - 4x_2 + 5x_3 + 8x_4 - 4x_5 = 0 \end{cases}$$

$$\Leftrightarrow \left[\begin{array}{ccccc|c} -3 & 6 & -1 & 1 & -7 & 0 \\ 1 & -2 & 2 & 3 & -1 & 0 \\ 2 & -4 & 5 & 8 & -4 & 0 \end{array} \right] \begin{array}{l} \uparrow +3 \\ \downarrow -2 \end{array}$$

$$\Leftrightarrow \left[\begin{array}{ccccc|c} 1 & -2 & 2 & 3 & -1 & 0 \\ 0 & 0 & 5 & 10 & -10 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array} \right] \begin{array}{l} \uparrow -5 \\ \downarrow \end{array}$$

$$\Leftrightarrow \left[\begin{array}{ccccc|c} 1 & -2 & 2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \uparrow -2 \\ \downarrow \end{array}$$

$$\Leftrightarrow \left[\begin{array}{ccccc|c} 1 & -2 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\Leftrightarrow \begin{cases} x_1 - 2x_2 - x_4 + 3x_5 = 0 \\ x_3 + 2x_4 - 2x_5 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x_1 = 2r + t - 3s \\ x_2 = r \\ x_3 = -2r + 2s \\ x_4 = t \\ x_5 = s \end{cases}$$

$$\Leftrightarrow x = r \underbrace{(2, 1, -2, 0, 0)}_u + t \underbrace{(1, 0, 0, 1, 0)}_v + s \underbrace{(-3, 0, 2, 0, 1)}_w$$

Jika $\text{Ker}(L) = \langle u, v, w \rangle$.

Selvestikin u, v, w ovat lin. riippomat (!!), joten

$$k = \dim \text{Ker}(L) = 3$$

Koska $k > 0$, niin L ei ole injektio.

Lisäksi $l = \dim L(\mathbb{R}^5) = 5 - 3 = 2 < \dim \mathbb{R}^3$,

joten L ei ole surjektiokaan.