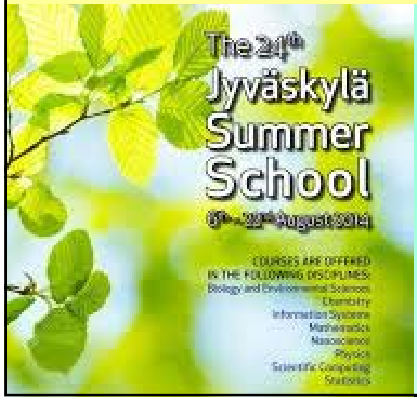
# "Properties of Nuclei deduced from the Nuclear Mass"




-the 2nd lecture-

SS Jyvaskyla
August 06-12, 2014


Yoshitaka Fujita
Osaka University

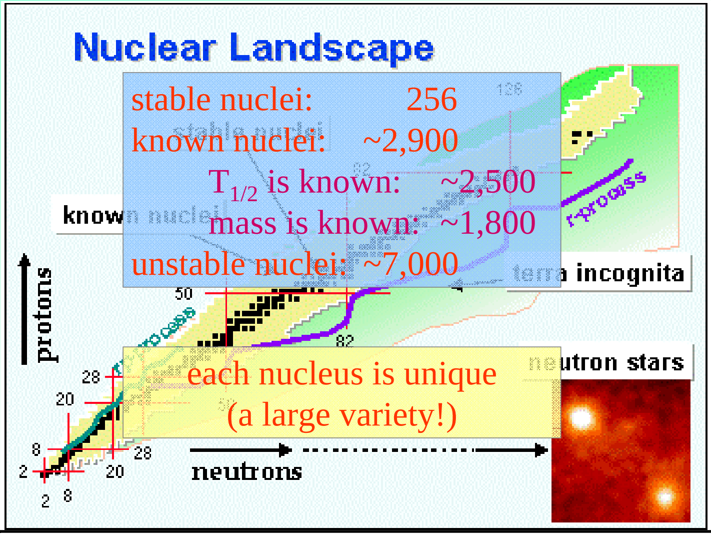

## Uniqueness of Nuclei

Nucleus : Unique Quantum System where
3 interactions out of 4 are active!

Strong, Weak, EM

(Gravitational force is too weak!)

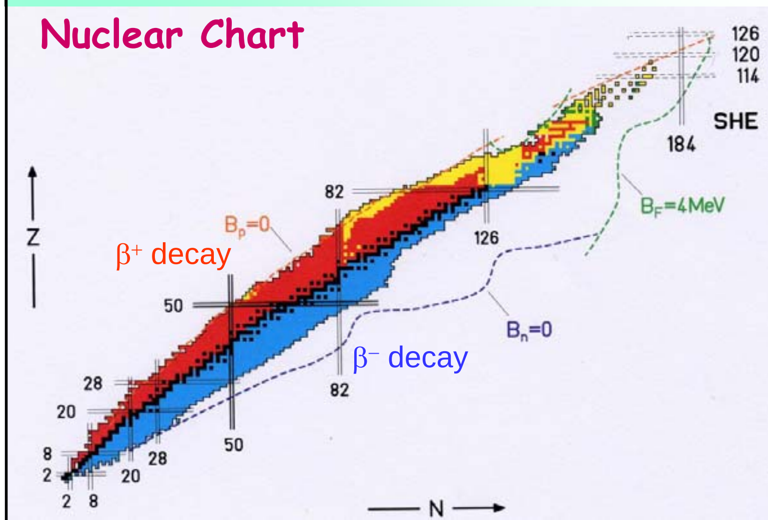
## Nuclear Chart



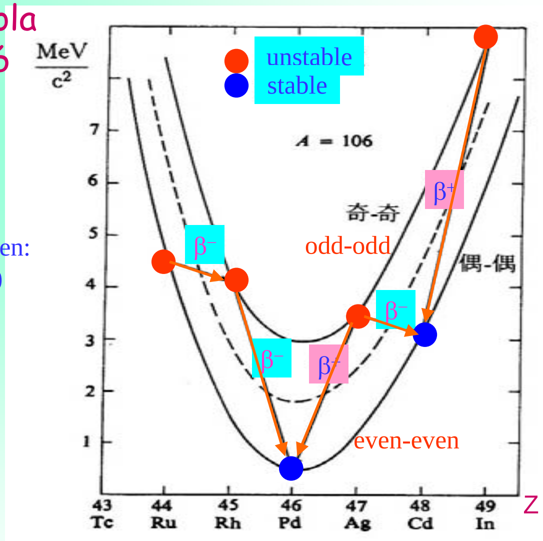

stable nuclei: 256
known nuclei: ~2,900
$T_{1/2}$ is known: ~2,500
mass is known: ~1,800
unstable nuclei: ~7,000

each nucleus is unique
(a large variety!)

## Nuclear Chart



## Mass Parabola (for A=106 Nuclei)




odd-odd & even-even: energies (masses) are different by $2\delta_0$

Pairing Int. is Important !

## Mass and Binding Energy of Nuclei

Bethe-Weizsäcker mass formula

Nuclear mass

$$m = Zm_p + Nm_n - \frac{E_B}{c^2}$$

Binding energy

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{\overbrace{(A-2Z)}^{=N-Z}{}^2}{A} + \delta(A,Z)$$

*mass term, surface term, Coulomb term, **symmetry term, pairing term (even-odd term)

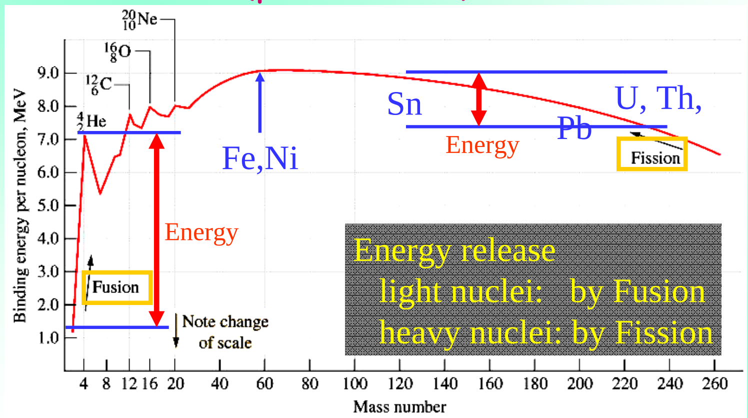
$$\delta(A,Z) = \begin{cases} +\delta_0 & Z, N \text{ even } (A \text{ even}) \\ 0 & A \text{ odd} \\ -\delta_0 & Z, N \text{ odd } (A \text{ even}) \end{cases} \qquad \delta_0 = \frac{a_P}{A^{1/2}}$$

* mass term shows that the nuclear force is short range!

**symmetry term originates from the Pauli exclusion principle for fermions!

## "mass" represents the overview!


## Nuclear Binding Energy (per nucleon)

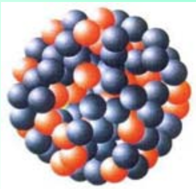


## Volume Surface Radius

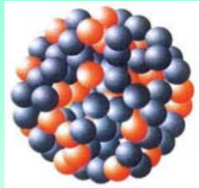
Nuclear Volume : proportional to mass A

Surface : $A^{2/3}$

Radius : $A^{1/3}$



## Coulomb Force: Combination of Protons



Permutation $_nP_r = \frac{n!}{(n-r)!}$

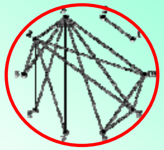
&

Combination $_nC_r = \frac{n!}{r!(n-r)!}$

The number of combination

→ r = 2 → $_nC_2$ = (1/2) n(n-1)

→ Coulomb int.: $-a_C \frac{Z(Z-1)}{A^{1/3}}$



Think of Coulomb Interaction among Protons

*Coulomb Interaction:
two-body interaction
a long range force

*— sign: repulsive

## Mass and Binding Energy of Nuclei

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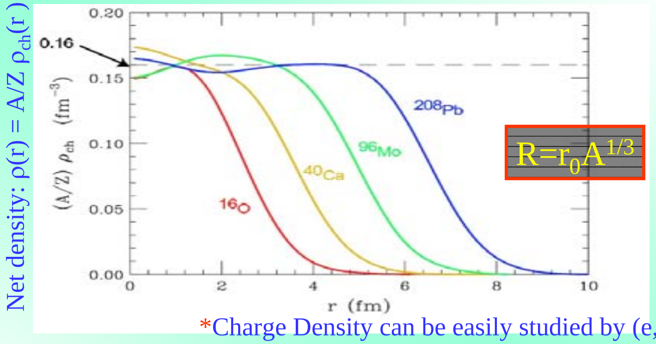
## Saturation of Nucleon Density in Nuclei

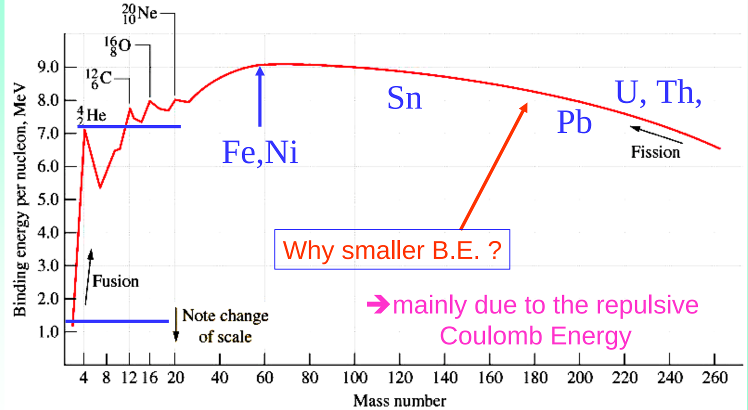



*Charge Density can be easily studied by (e,e')

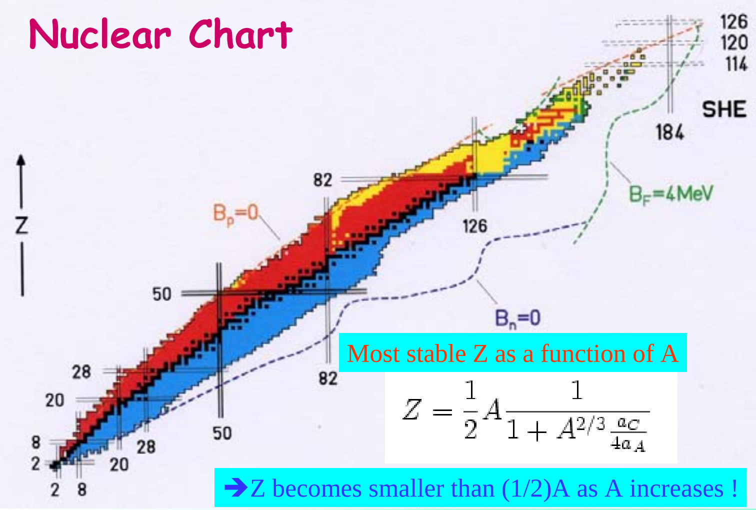
➔due to the "short range" nature of nuclear interaction
➔due to the intermediate mass of pion (~135 MeV)

➔therefore, the two-body Nucleon-Nucleon int. is dominant !
➔therefore, the "mass term" is proportional to mass number A

## Nuclear Binding Energy
### -Overview-




## Nuclear Chart




Most stable Z as a function of A

$$Z = \frac{1}{2} A \frac{1}{1 + A^{2/3}\frac{a_C}{4a_A}}$$

➔Z becomes smaller than (1/2)A as A increases !

## Mass and Binding Energy of Nuclei

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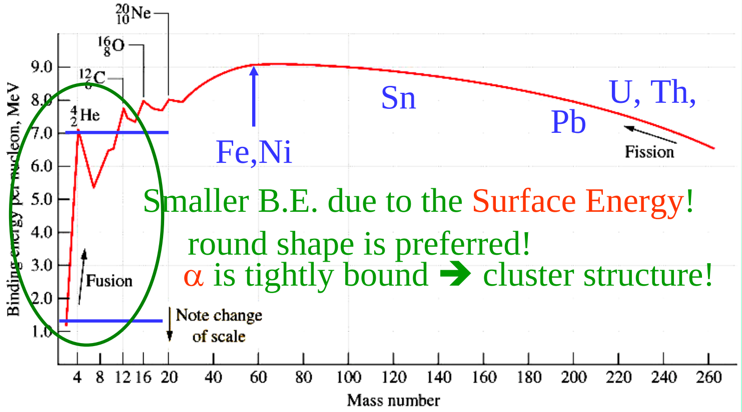
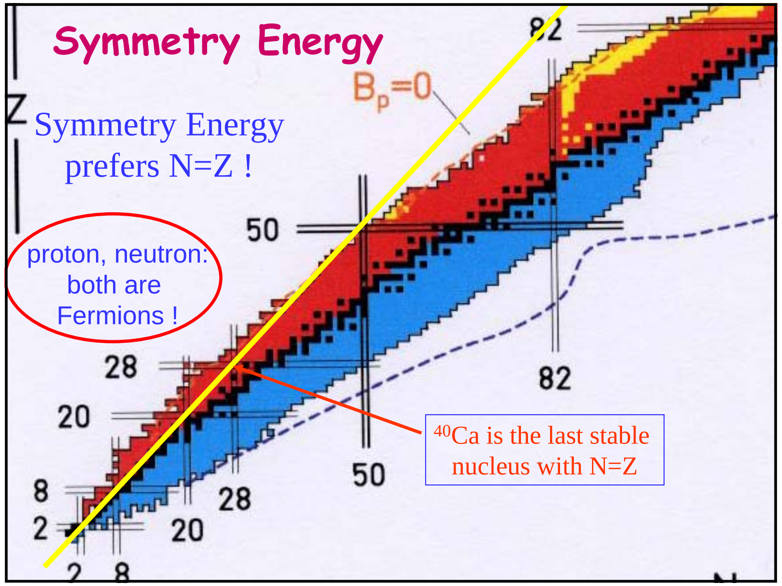
## Nuclear Binding Energy
### -Surface Energy-




## Symmetry Energy

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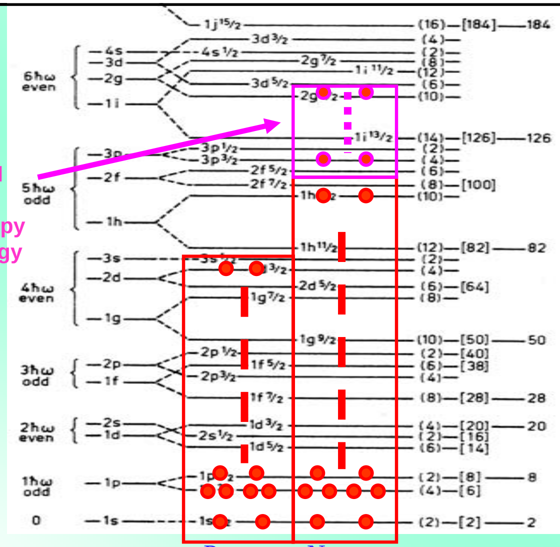
## Filling of Shells

Additional neutrons would occupy higher-energy shells !




Protons Neutrons

# Mass and Binding Energy of Nuclei

Nuclear mass

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Bethe-Weizsäcker mass formula

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# Nuclear Chart

for each A only one nucleus is stable

at most 2

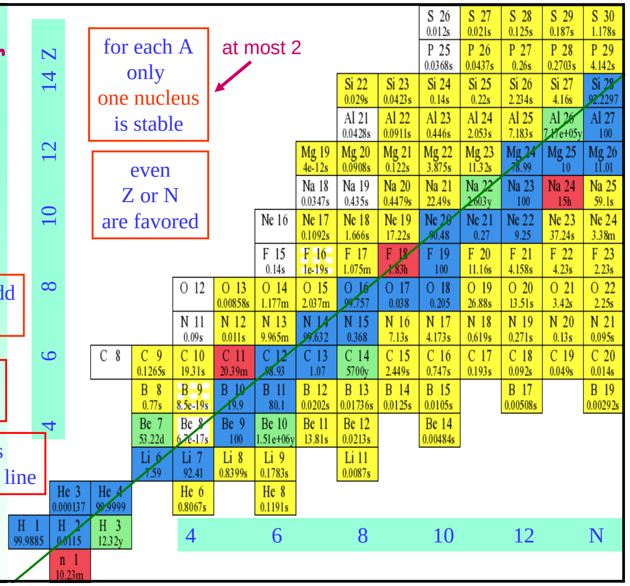
even Z or N are favored

rare !

stable odd-odd $^{10}B$, $^{14}N$

A=5, 8 are missing!

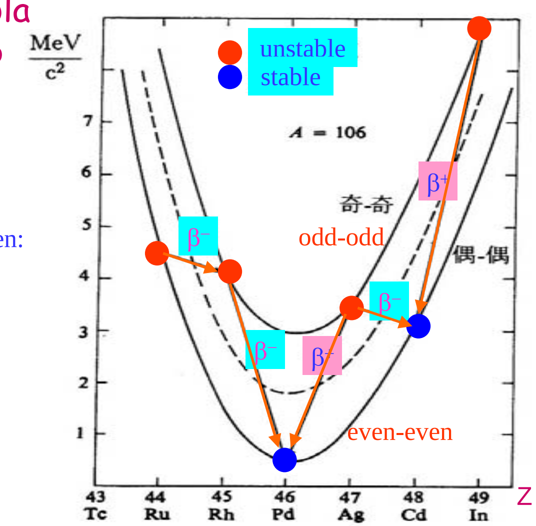
Only $^3He$ is above the N=Z line



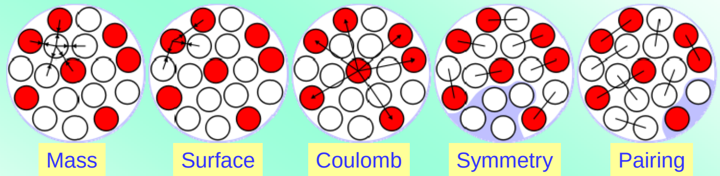

# Mass Parabola (for A=106 Nuclei)

odd-odd & even-even: energies (masses) are different by $2\delta_0$

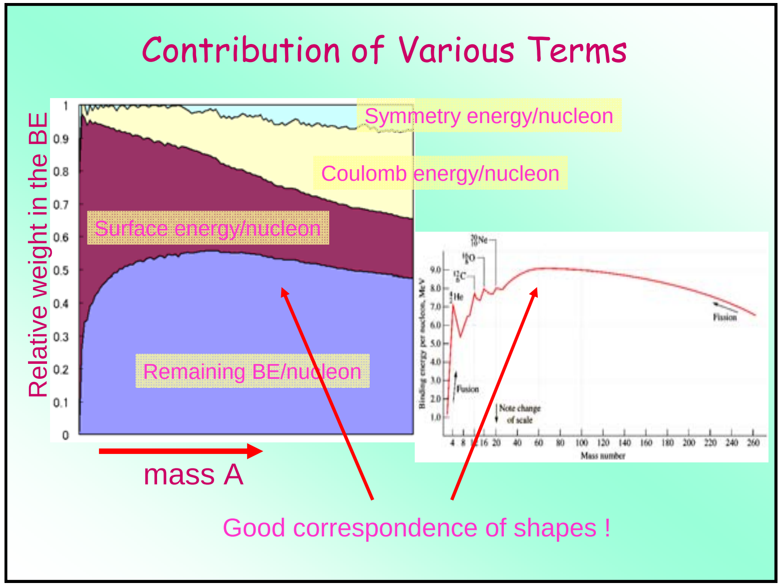
Pairing Int. is Important !

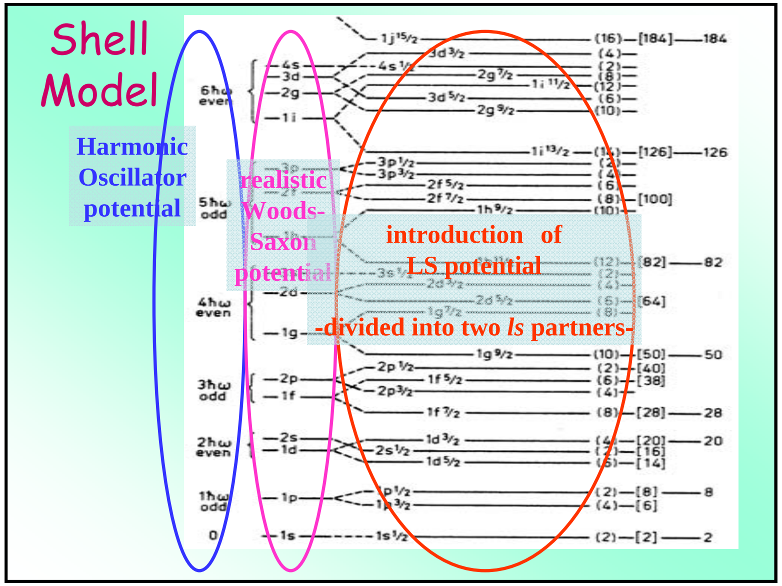


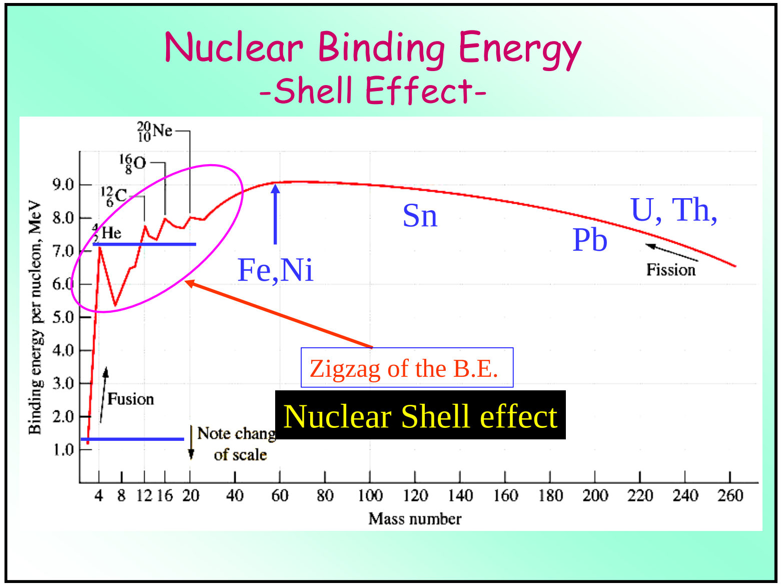

# Terms in Mass Formula and Interactions (Correlations) in Nuclei

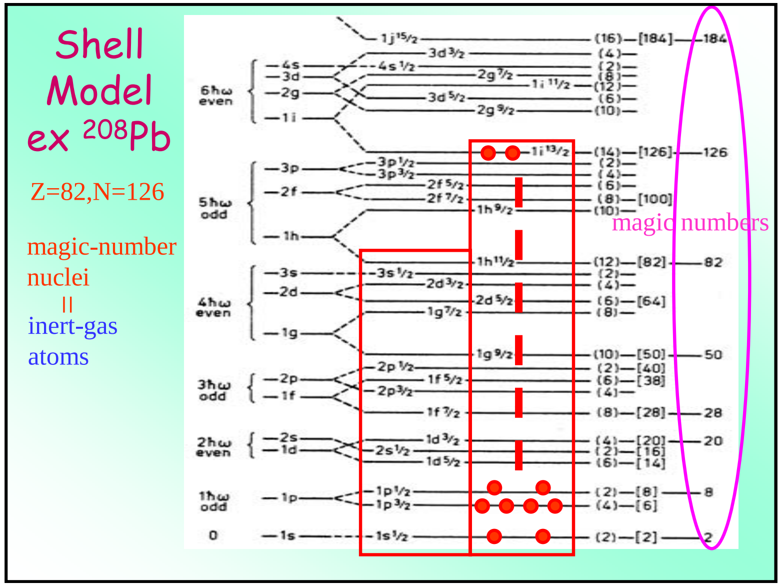


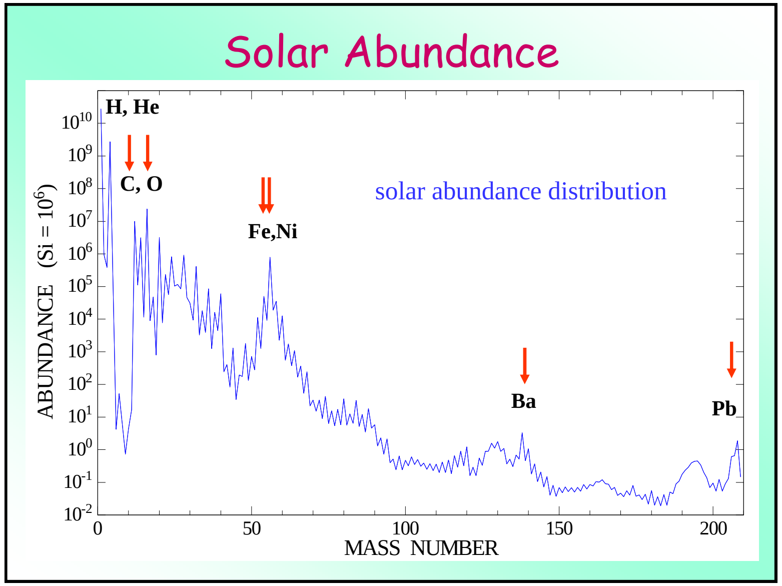

| Mass / Surface | Coulomb | Symmetry | Pairing |
|---|---|---|---|
| Main part of the Nuclear Interaction is short range ! | Coulomb Interaction is Strongly | p-n Interaction is Important. | p-p, n-n Interactions are Important. |
| Main part of the NI is Attractive | Coulomb force is Repulsive. | p-n int. is Attractive | p-p, n-n int. is Attractive |

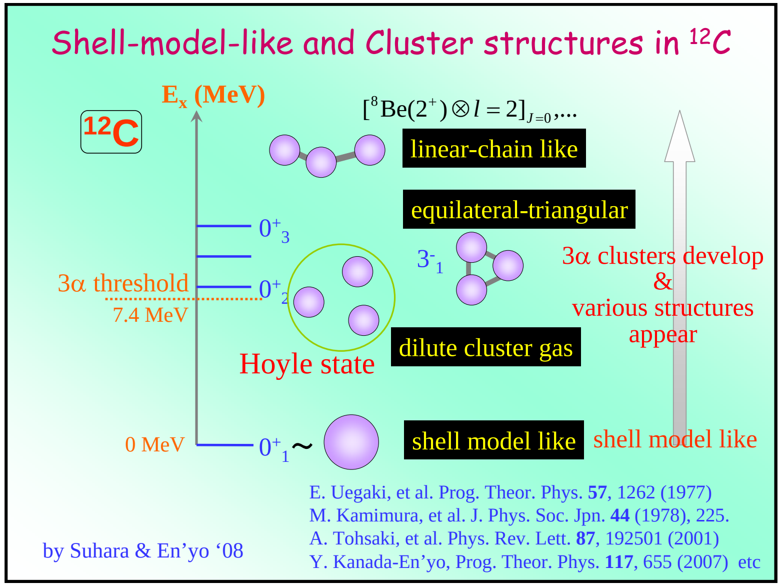

Contribution of Various Terms
Symmetry energy/nucleon
Coulomb energy/nucleon
Surface energy/nucleon
Remaining BE/nucleon
Relative weight in the BE
mass A
Good correspondence of shapes !

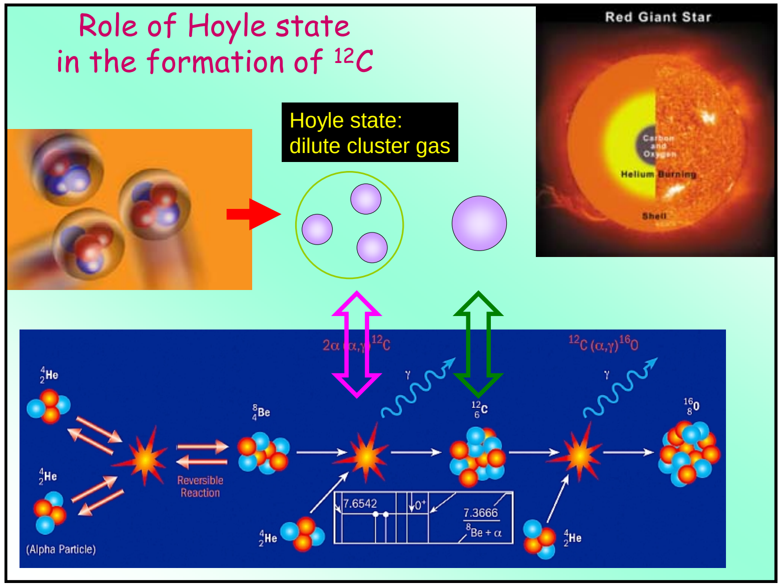

Shell Model
Harmonic Oscillator potential
realistic Woods-Saxon potential
introduction of LS potential
-divided into two ls partners-

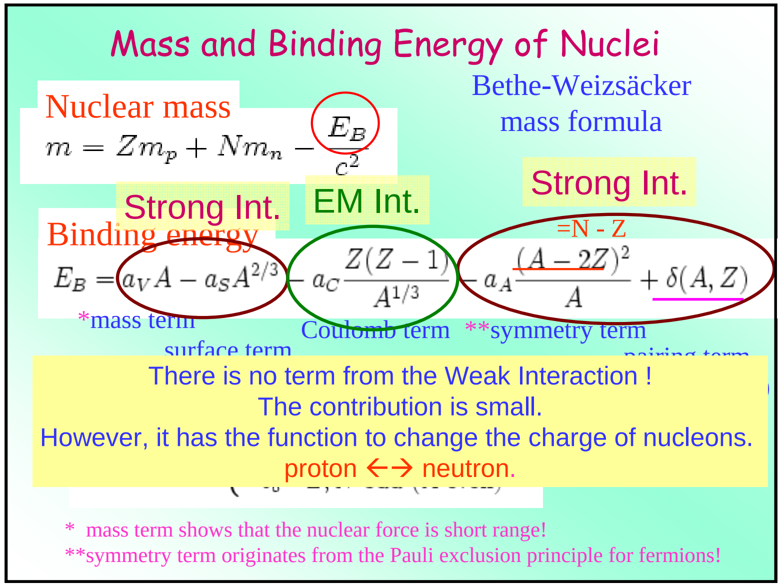

Nuclear Binding Energy
-Shell Effect-
Binding energy per nucleon, MeV
Mass number
Fusion
Fission
Fe,Ni
Sn
Pb
U, Th,
Zigzag of the B.E.
Nuclear Shell effect


Shell Model
ex 208Pb
Z=82,N=126
magic-number nuclei
||
inert-gas atoms
magic numbers


Solar Abundance
H, He
C, O
Fe,Ni
solar abundance distribution
Ba
Pb
ABUNDANCE (Si = 10^6)
MASS NUMBER


Shell-model-like and Cluster structures in 12C
12C
Ex (MeV)
[8Be(2+)⊗l = 2]J=0,...
linear-chain like
equilateral-triangular
3-1
3α clusters develop
&
various structures
appear
0+3
3α threshold
7.4 MeV
0+2
Hoyle state
dilute cluster gas
0 MeV
0+1 ~
shell model like
shell model like
E. Uegaki, et al. Prog. Theor. Phys. 57, 1262 (1977)
M. Kamimura, et al. J. Phys. Soc. Jpn. 44 (1978), 225.
A. Tohsaki, et al. Phys. Rev. Lett. 87, 192501 (2001)
Y. Kanada-En'yo, Prog. Theor. Phys. 117, 655 (2007) etc
by Suhara & En'yo '08


Role of Hoyle state
in the formation of 12C
Hoyle state:
dilute cluster gas
Red Giant Star
Helium Burning
Shell
2α(α,γ)12C
12C(α,γ)16O
Reversible Reaction
7.6542
7.3666
(Alpha Particle)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Bethe-Weizsäcker
mass formula
Strong Int.
EM Int.
Strong Int.
Binding energy
=N - Z
$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A,Z)$
*mass term
surface term
Coulomb term
**symmetry term
There is no term from the Weak Interaction !
The contribution is small.
However, it has the function to change the charge of nucleons.
proton ←→ neutron.
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**symmetry term originates from the Pauli exclusion principle for fermions!