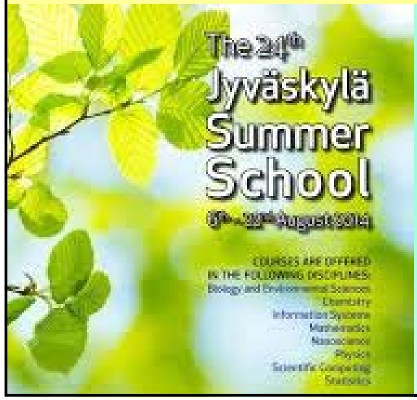
Overview of Nuclear Structure and Excitations

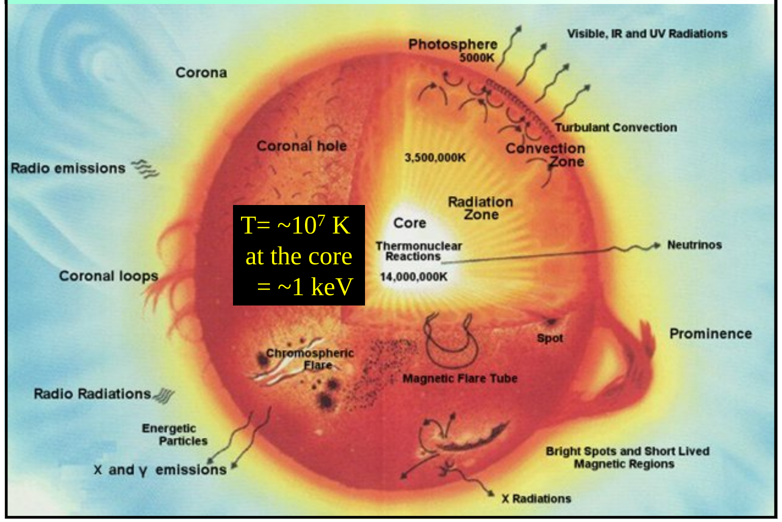


-the 3rd lecture-

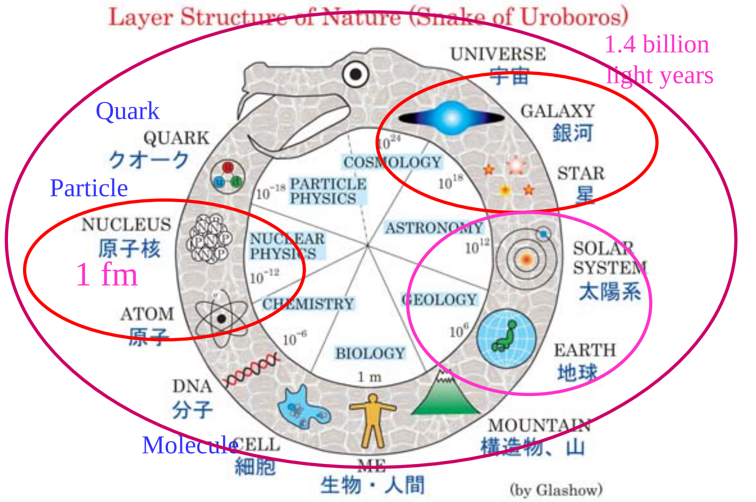
SS Jyvaskyla
August 06-12, 2014

Yoshitaka Fujita
Osaka University

MAJOR FEATURES OF OUR SUN



自然の階層性（ウロボロスの蛇）



^{1}H (p) → ^{4}He: Nuclear Reaction in a Star

$p + p \rightarrow {}^2H + e^+ + \nu_e \quad (\nu_{pp})$

弱い相互作用が関与する β−崩壊
(Weak processes control the reaction rate!)

$p + {}^2H \rightarrow {}^3He + \gamma$

86 %: ${}^3He + {}^3He \rightarrow {}^4He + 2p$

14 %: ${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

14 %: ${}^7Be + e^- \rightarrow {}^7Li + \nu_e \quad (\nu_{Be})$

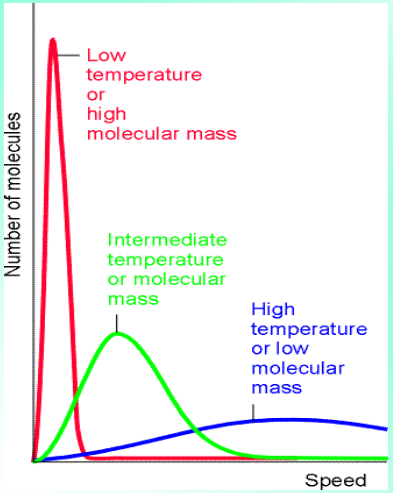
$p + {}^7Li \rightarrow {}^4He + {}^4He$

0,02 %: ${}^7Be + p \rightarrow {}^8B + \gamma$

${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e \quad (\nu_B)$

${}^8Be^* \rightarrow {}^4He + {}^4He$

Maxwell distribution



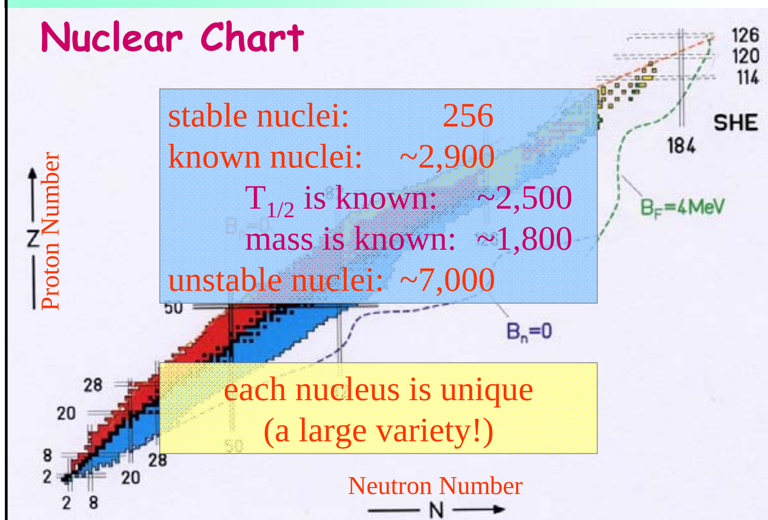
velocity
$\langle v^2 \rangle = 3kT/m$
kinetic energy
$\langle K \rangle = 3kT/2$

10^4K~ 1 eV
10^7K~ 1 keV
10^9K~100 keV

How do we see nuclei?
How do we see Table Mountain?

***How Do We See Nuclei?

Nuclear Chart



stable nuclei: 256
known nuclei: ~2,900
$T_{1/2}$ is known: ~2,500
mass is known: ~1,800
unstable nuclei: ~7,000

each nucleus is unique
(a large variety!)

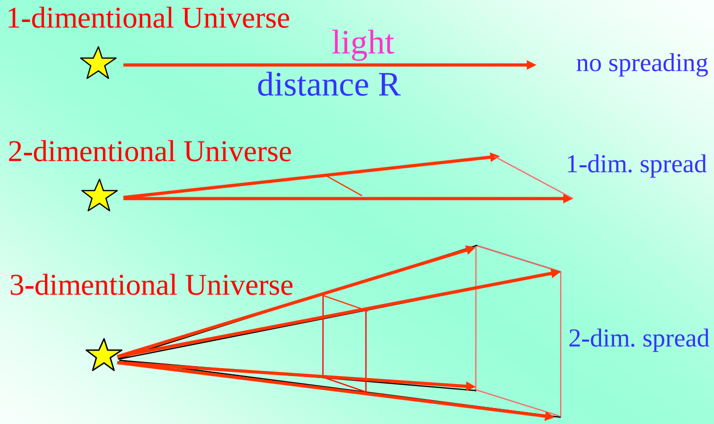
Uniqueness of Nuclei

Nucleus : Unique Quantum System where
3 interactions out of 4 are active!

Strong, Weak, EM

(Gravitational force is too weak!)

Long and short-range forces



brightness of a star $\propto 1/R^2$

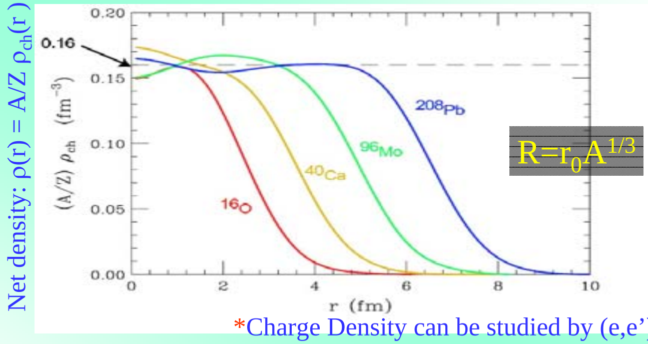
i.e., inversely prop. to the expansion of the space

4-fundamental interactions (forces)

Interactions	Example	Transmitter
Strong int.*	Nuclear Force	Meson* (Gluon)
EM int.#	Coulomb Force	Photon#
Weak int.*	Beta-decay	W-, Z-boson*
Gravitational#	Apple falls!	Graviton#

*short-range #long-range *with mass #mass-less

Saturation of Nucleon Density in Nuclei



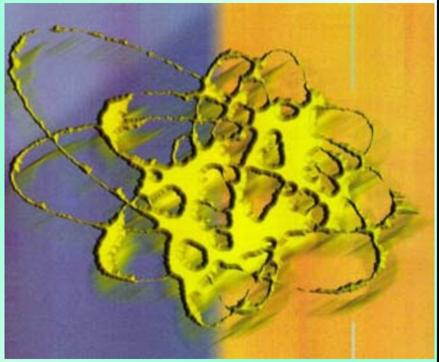
*Charge Density can be studied by (e,e')

➔due to the "short range" nature of nuclear interaction
➔due to the intermediate mass of pion (~135 MeV)

➔therefore, **Two body Nucleon-Nucleon int.** is dominant !

Image of Nuclei

Saturation of
Nuclear Density!



"Nuclear Physics" by Bohr and Mottelson Cover page (1969)

How Nuclei are defined ?

*Quantum Finite Many-body System of Fermions

=> quantum numbers are important

L, S, J, K, T

=> selection rules of Q-numbers are importnat

*Active forces in nuclei:

3 out of 4 fundamental forces

strength: **strong** >> **electro-magnetic** >> **weak**

time :	fast	middle	slow
	(~10^{-20}s)	(~10^{-15}s)	(~10^{-1}s)

*they struggle to make their territory larger !

→phenomena from 3 forces can be combined for the study of nuclei !

How are Nuclei defined ?

*Quantum Finite Many-body System of Fermions

=> quantum numbers are important

L, S, J, K, T

=> selection rules of Q-numbers are important

*Conservation Laws

Energy

Momentum

Angular momentum

*Studied by Measuring Decays, Reactions

γ-decay, β-decay, Nuclear Reactions

Roles of 3 forces

in Nuclear excitation & decay

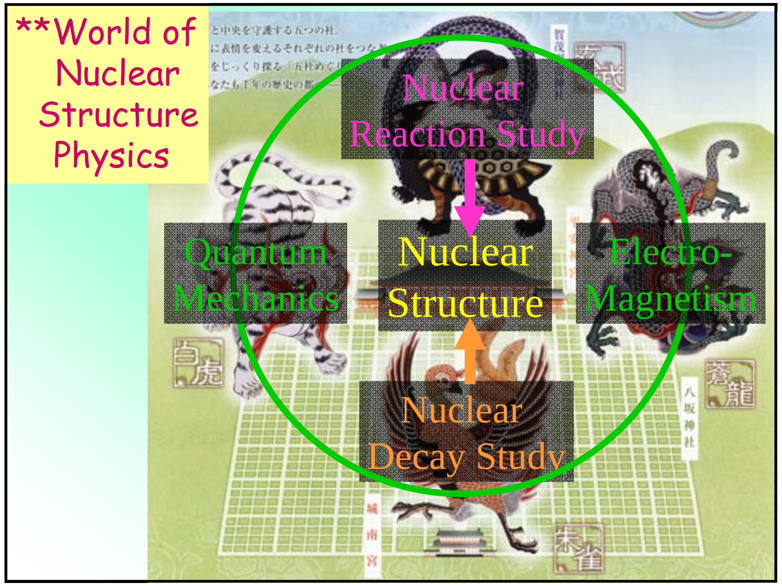
Strong:	nuclear reactions
	[(p, p'), (α, α'),.., (p, n), (^{3}He,t) etc]
EM:	(e, e'), Coulomb ex., γ-decay
Weak:	ν-induced reactions, β-decay
	[(ν_x, ν_x'), (ν_e, e^-),…]

*if Strong can play a role, other two are hidden!

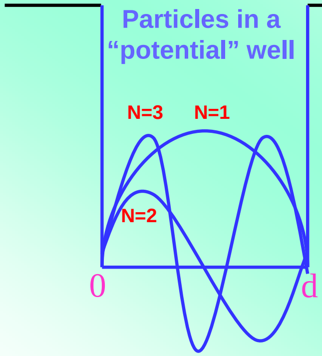
*if EM , Weak

*if Strong and EM cannot play roles,

then Weak will appear on the stage.



Nuclei: Quantum system

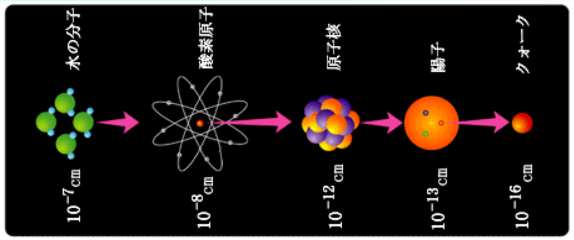


Wave nature of particles and **Confinement** is the origin of **Quantized energies** and **Levels** !

for confinement
$N\ \lambda/2 = d$
using de Broglie relationship
$p = h/\lambda = hN/2d$
then
$E = p^2/2m = N^2h^2/(8md^2)$
+ zero point energy

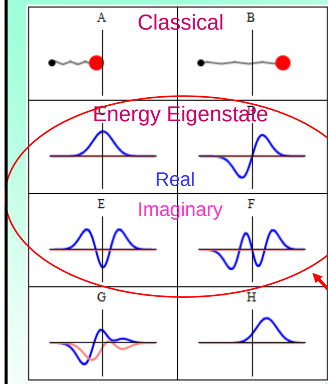
N = 1,2,3 is the principal quantum number
E goes ↗ with **N**, because the wave length is shorter

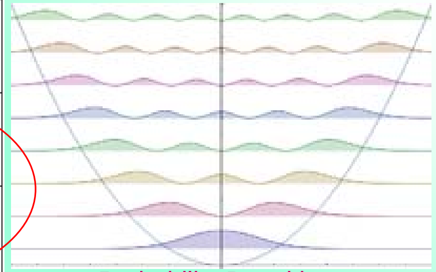
Uniqueness of Nuclei



Nucleus: Quantum Finite System
freedom: protons Z & neutrons N
further freedom: mesons
further more freedom: quarks
Shell model

Harmonic Oscillator





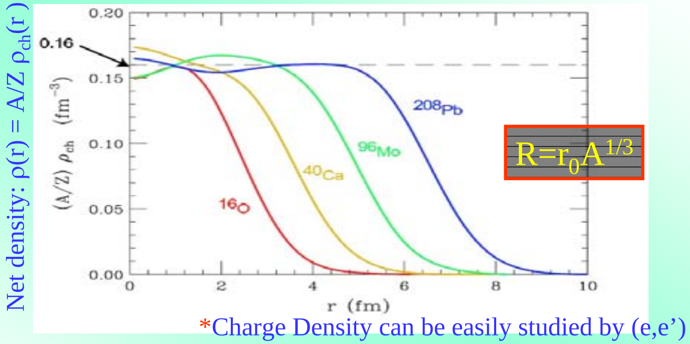
Probability Densities
Quantum **n**-phonon Oscillation

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right).$$

from Wikipedia

Saturation of Nucleon Density in Nuclei

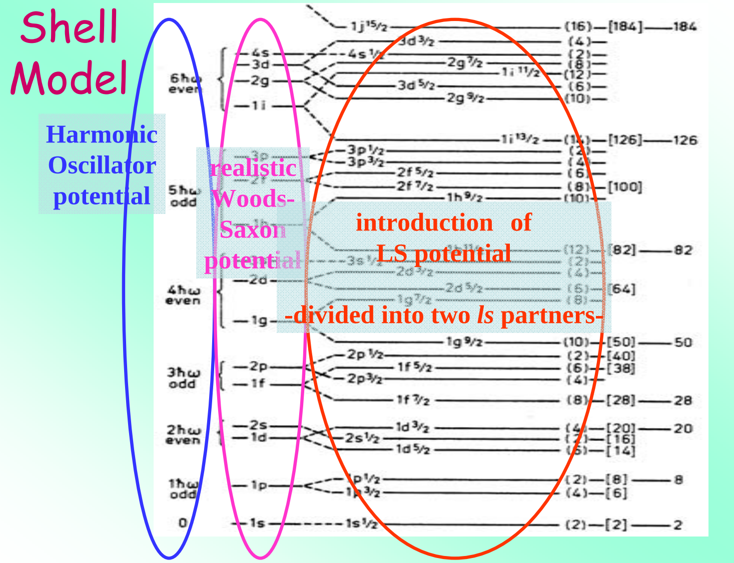


*Charge Density can be easily studied by (e,e')

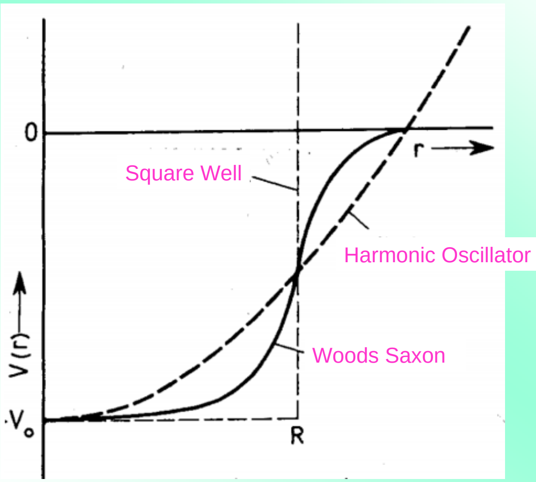
➔due to the "short range" nature of nuclear interaction
➔due to the intermediate mass of pion (~135 MeV)

➔therefore, the two-body Nucleon-Nucleon int. is dominant !
➔therefore, the "mass term" is proportional to mass number A

Shell Model



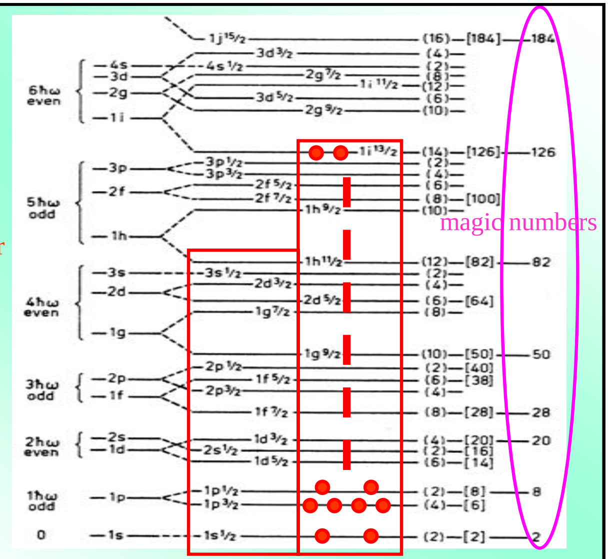
Nuclear Potentials



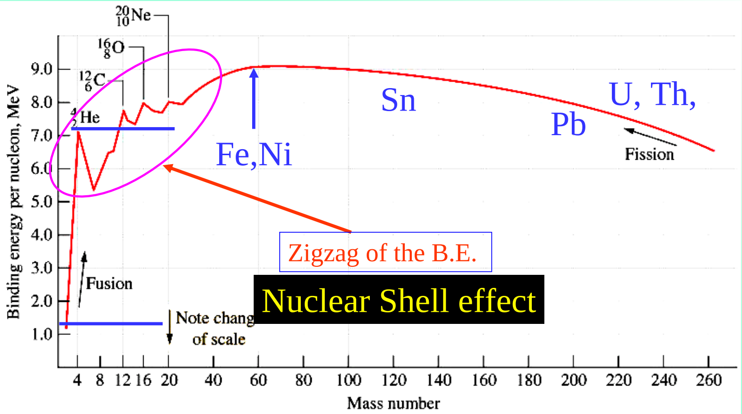
Shell Model ex ^{208}Pb

Z=82,N=126

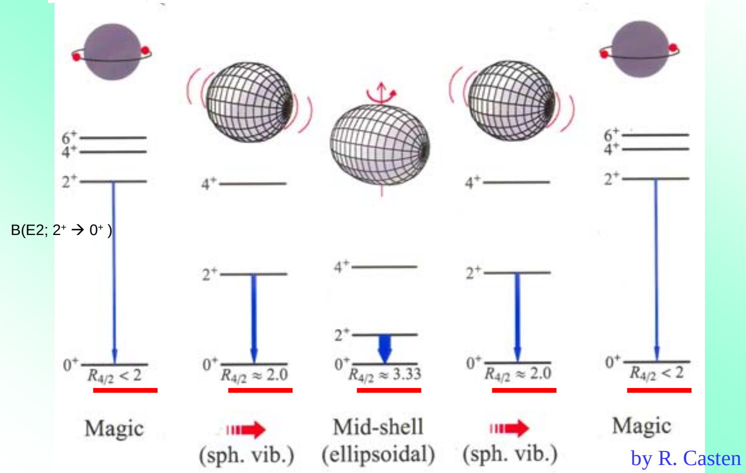
magic-number nuclei
||
inert-gas atoms



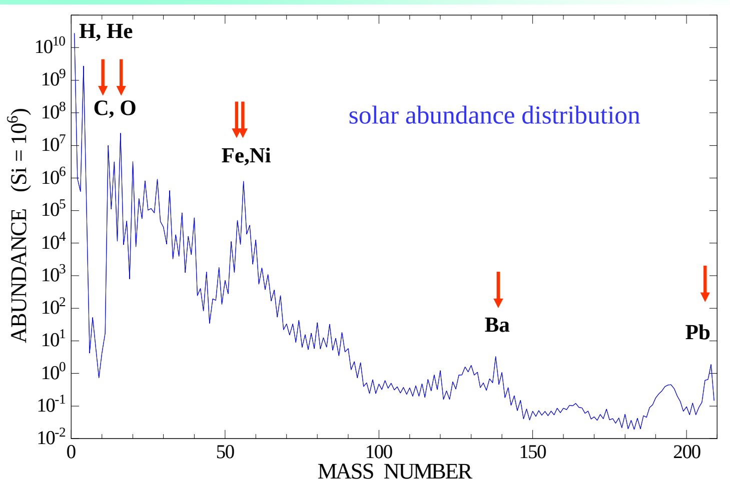
Nuclear Binding Energy
-Shell Effect-



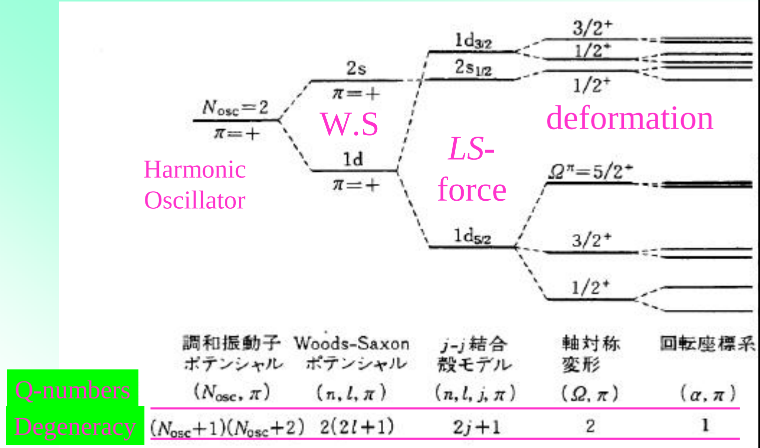
Evolution of Nuclear Structure
- low-lying states between two shell closures -



Solar Abundance



Degeneracy of single particle states



Note on SM & "Residual Interactions" (I)

In the Shell-Model nucleons are treated as independent.

Single-particle phenomena are usually well described.

ex. J^{π} values of ground states of odd nuclei

Nucleus	J^{π}
${}^{3}_{2}He_{1}$	$1/2^{+}$
${}^{7}_{3}Li_{4}$	$3/2^{-}$
${}^{17}_{8}O_{9}$	$5/2^{+}$
${}^{41}_{20}Ca_{21}$	$7/2^{-}$

Shells deep inside are treated as inert.

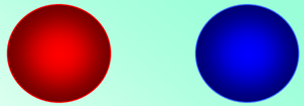
Doubly magic nuclei form "inert core".

ex. ${}^{4}He$ (N=Z=2), ${}^{16}O$ (N=Z=8), ${}^{40}Ca$ (N=Z=20), ${}^{56}Ni$ (N=Z=28)

Nucleon & Coin

back face

= Coin



proton neutron

= Nucleon

similar mass

nearly the same interaction

$T_z=-1/2$ $T_z= 1/2$ isospin T=1/2

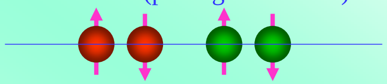
Note on SM & "Residual Interactions" (II)

However, J^{π} values of even-even nuclei are $J^{\pi}=0^{+}$.

→ We notice the importance of the spin-spin coupling.

(pairing interaction)

ex. $1s_{1/2}$



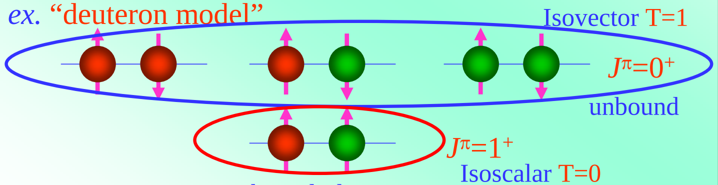
proton neutron

$={}^{4}He = \alpha$

$J^{\pi}=0^{+}$

In general, interactions that are not included in a model are called "residual interactions"

ex. "deuteron model"



Isovector T=1

$J^{\pi}=0^{+}$

unbound

$J^{\pi}=1^{+}$

Isoscalar T=0

bound=deuteron

***Observing Residual Interactions

-e.g. Coupled Pendulum -

Note on Nuclear Model & "Residual Interactions"

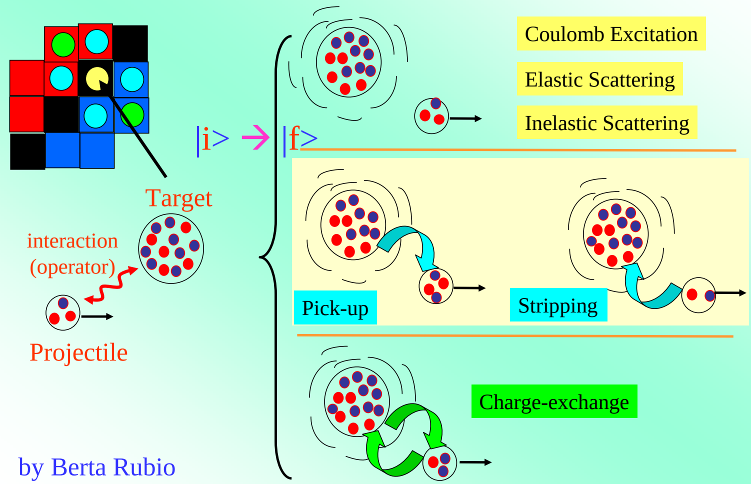
We first assume a nuclear model.
ex. Harmonic Oscillator Model,
or Shell Model.

Remaining part of nucleon-nucleon interactions that are not included in the model are treated as "residual (or effective) interactions."

Residual interactions between valence nucleons play important roles to form nuclear structure.

Mainly 2-body int., but also 3-body int.

Direct Reactions with Light Projectiles



by Berta Rubio

** "Nuclear Excitations" in Nuclei

Structure information form Transitions

Nuclear Transitions give us Structure information

*Transition strength: proportional to $|\langle f|\, \boldsymbol{Op}\, |i\rangle|^2$

$$\mathbf{H}_i|i\rangle = E_i|i\rangle, \quad \mathbf{H}_f|f\rangle = E_f|f\rangle$$

*Studied by: Nuclear Reactions, Decays
Reaction: Excitation + Spectroscopy
Decay: Spectroscopy

*Mode of Excitation ←→ ***Op***

For the study of Nuclear Structure

We have two different tools!

1) Decay Studies

γ-decay: in beam γ-study, source study

β-decay: β-ray study, β-delayed γ, ***p*** or ***n***

2) Reaction Studies

Inelastic Scattering: simply giving Energy

Charge Exchange Reaction:

charge-exchange & giving Energy

Pick-up Reaction, Transfer Reaction, …

***Observation of Nuclear Excitations

Nuclear Reaction and Decay Studies

Nuclear Reaction

In-coming particle with E_{in}

Out-going particle with E_{out}

*out-going particles are analyzed

by energy

by angular distribution…

Decay Studies

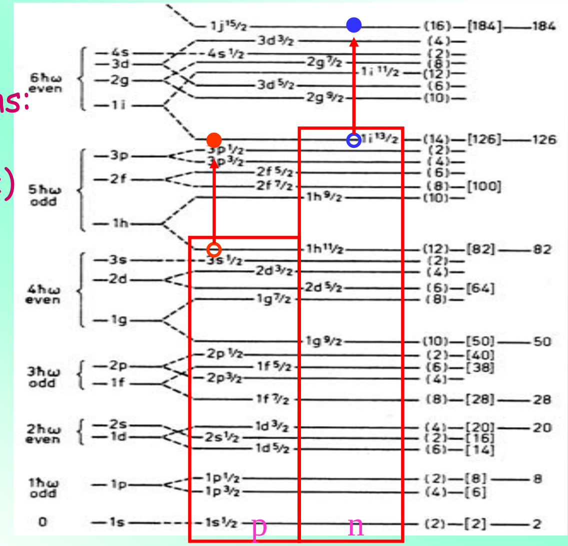
β decay, γ decay, particle decay

*unstable nuclei are first produced

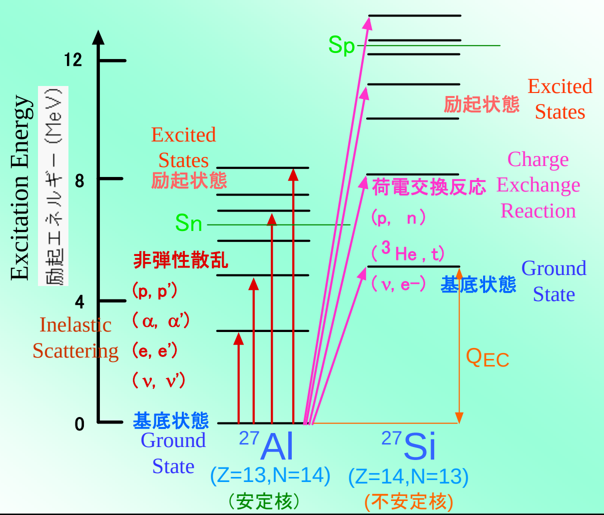
*then decays are measured

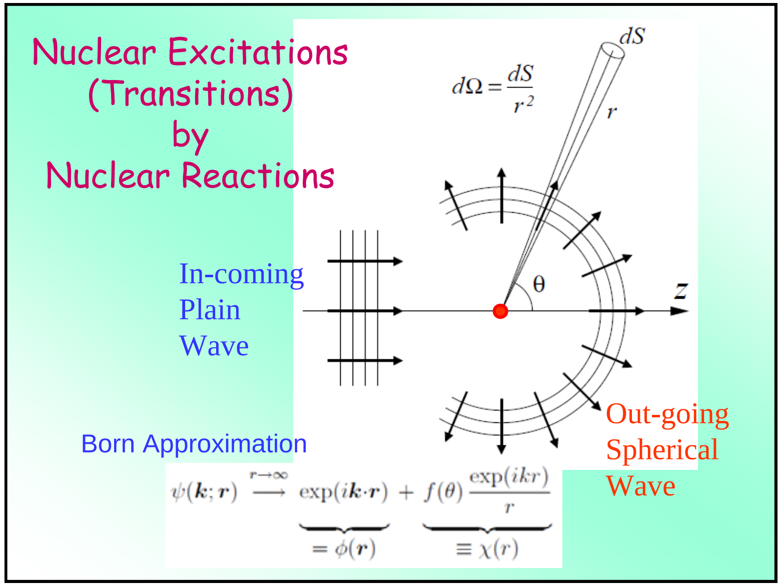
Basic Excitations: 1p-1h (inelastic)

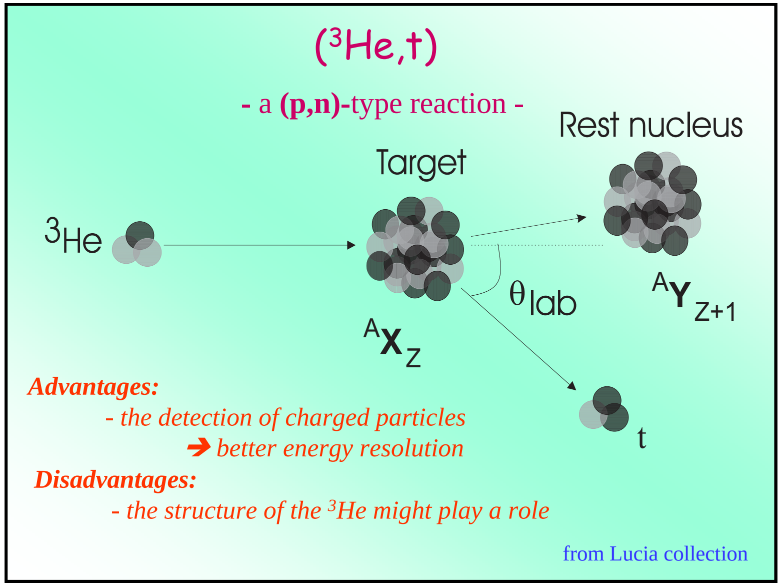
Z=82,N=126

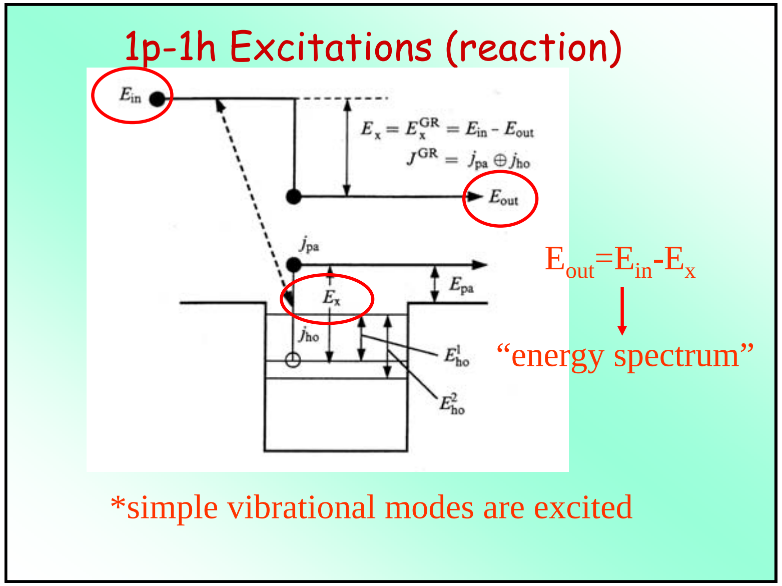


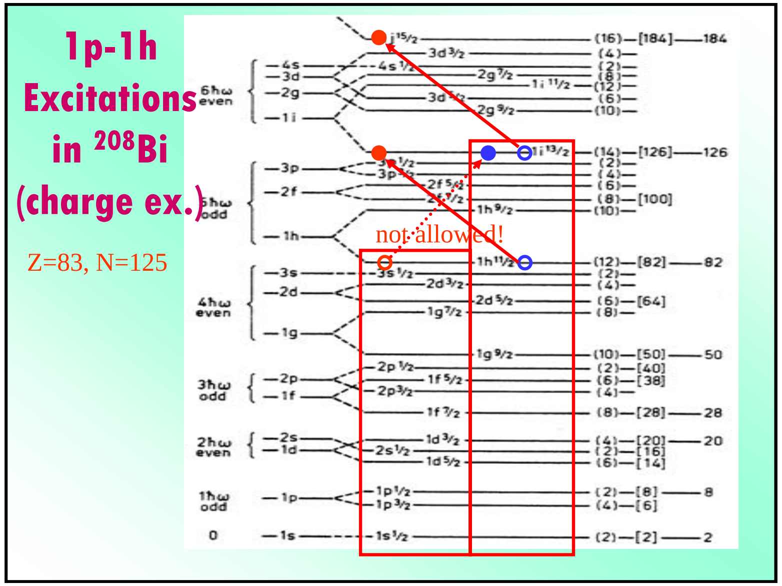
Nuclear Excitations by Reactions




Nuclear Excitations
(Transitions)
by
Nuclear Reactions
$d\Omega = \frac{dS}{r^2}$
dS
r
θ
z
In-coming
Plain
Wave
Out-going
Spherical
Wave
Born Approximation
$\psi(\boldsymbol{k};\boldsymbol{r}) \xrightarrow{r\to\infty} \underbrace{\exp(i\boldsymbol{k}\cdot\boldsymbol{r})}_{=\phi(\boldsymbol{r})} + \underbrace{f(\theta)\frac{\exp(ikr)}{r}}_{\equiv\chi(r)}$

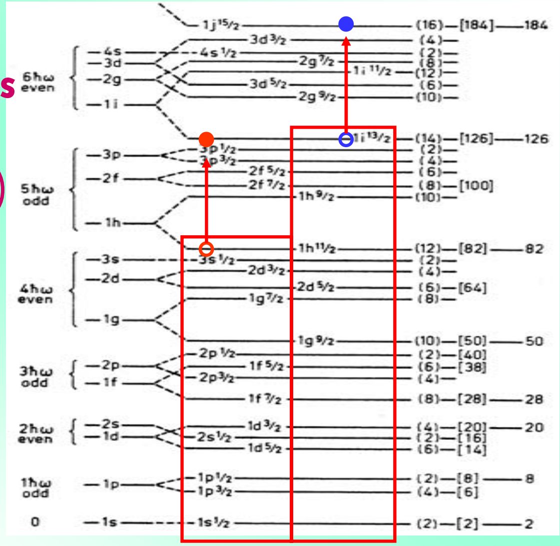

(^{3}He,t)
- a (p,n)-type reaction -
Rest nucleus
Target
^{3}He
θlab
$^AY_{Z+1}$
^AX_Z
t
Advantages:
- the detection of charged particles
→ better energy resolution
Disadvantages:
- the structure of the ^{3}He might play a role
from Lucia collection


1p-1h Excitations (reaction)
E_{in}
$E_x = E_x^{GR} = E_{in} - E_{out}$
$J^{GR} = j_{pa} \oplus j_{ho}$
E_{out}
j_{pa}
E_x
E_{pa}
j_{ho}
E_{ho}^1
E_{ho}^2
$E_{out}=E_{in}-E_x$
"energy spectrum"
*simple vibrational modes are excited

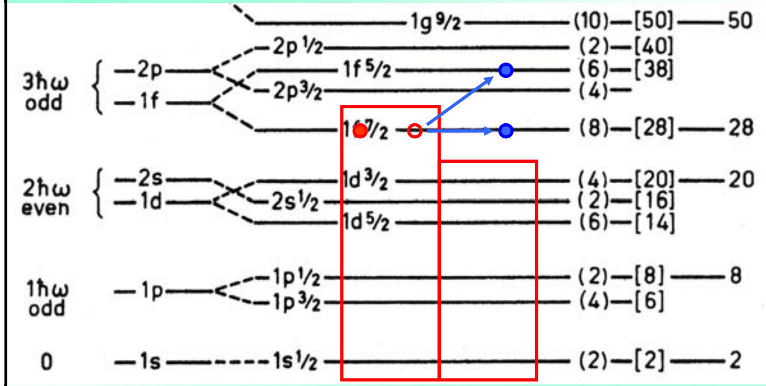

1p-1h
Excitations
in ^{208}Bi
(charge ex.)
Z=83, N=125
not allowed!
126
82
50
28
20
8
2
184

1p-1h Excitations in ^{208}Pb (inelastic)

Z=82,N=126



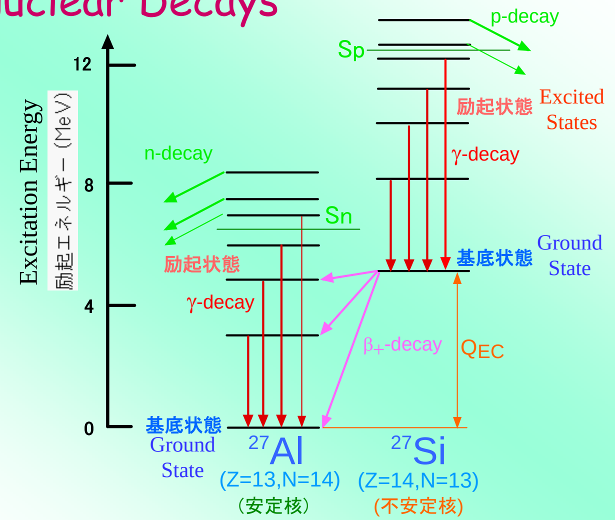
Transitions from ^{42}Ti : β decay



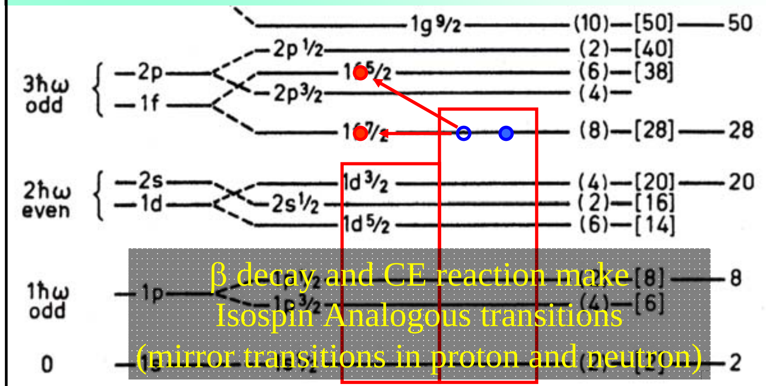
proton: $f_{7/2}$ → neutron $f_{7/2}$

proton: $f_{7/2}$ → neutron $f_{5/2}$

Nuclear Decays



Transitions from ^{42}Ca : CE Reaction



neutron: $f_{7/2}$ → proton $f_{7/2}$

neutron: $f_{7/2}$ → proton $f_{5/2}$

What do we observe?

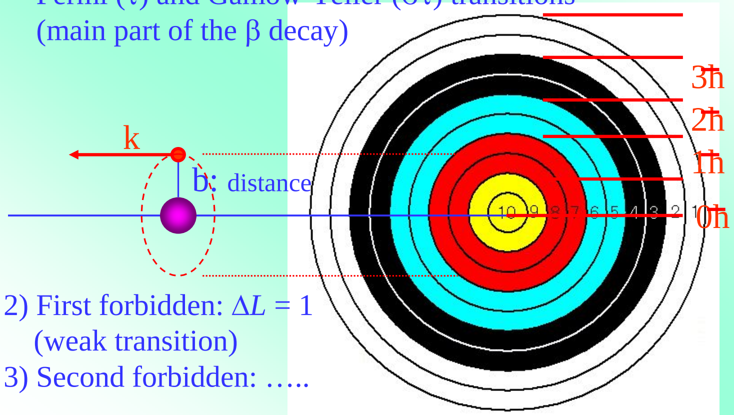
Observed Strength
= reaction mechanism
⊗ operator
⊗ structure
* integration of 3-quantities!

	mechanism	operator (interaction)
γ decay:	$\boxed{\text{simple}}$	EM
β decay:	$\boxed{\text{simple}}$	weak
reaction:	complicated	strong

*to study Structures, other 2 should be simple!

β-decay

1) Allowed β decay: $\Delta L = 0$
Fermi (τ) and Gamow-Teller ($\sigma\tau$) transitions
(main part of the β decay)



2) First forbidden: $\Delta L = 1$
(weak transition)
3) Second forbidden: …..

Case 1 : γ-decay & β-decay

*both have very simple mechanism.
(people even don't think of "mechanism !")

*Operators are relatively simple!
Weak : Gamow-Teller, Fermi
EM : E1, E2,… M1, M2,…

matrix element & $t_{1/2}$

$(1 / t_{1/2})$ = Coup.Const. x PhaseSpaceFac.
x $|\langle \mathbf{f} \mid \boldsymbol{Op} \mid \mathbf{i} \rangle|^2$

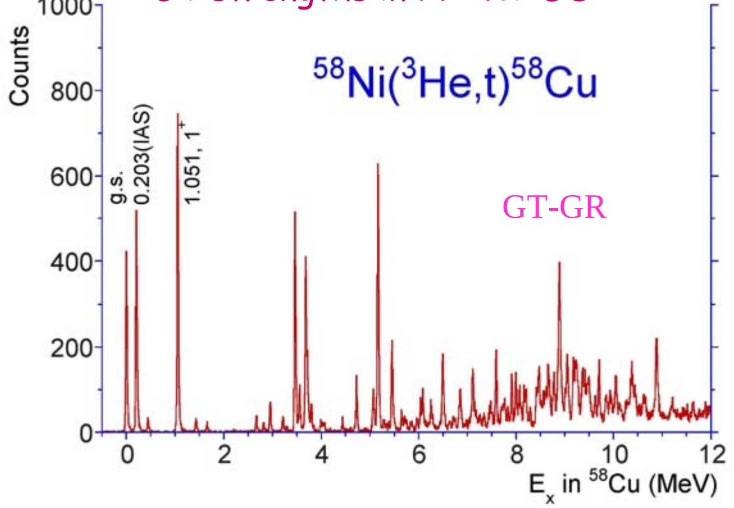
*if *Op* is specified, w.f.(=structures) are studied !
(*Op* specification is not always easy!)
* highly *Ex* region cannot be reached !

Reduced transition strength *B* (*Op*)

A value proportional to (matrix element)^2
$|\langle \mathbf{f} \mid \boldsymbol{Op} \mid \mathbf{i} \rangle|^2$
is called "reduced transition strength"
ex. *B*(GT), *B*(F), *B*(*M1*), *B*(*E2*),….

*representing only the structure part
for a specific operator!
*reaction mechanism part is removed!

GT strengths in A=42-58



***Operators = Hammers ??
Nucleus = Bell ??

Case 2 : Study by Nuclear Reactions

*we have to think of mechanisms seriously.
1) one-step, two-step,…
2) direct, exchange

*we have to think of operators (modes) seriously.
*separation of excitation modes is the main subject.
various reactions (e.g. charge exch., inelastic,..)
using different particles,
at different incident energies.
angular distribution analysis (*"L"* analysis)

*complicated, but highly *Ex* region can be reached!
(reaction study is a dirty business, but effective!)

Todaiji Temple in Nara

Hit a Bell ! Hit a Nucleus!

at Todaiji temple
Nara, Japan

from Lucia collection

Various Reaction Mechanism / How and Where you hit the bell/

how and where you hit
=reaction mechanism

The sound from the bell is different how and where you hit!

The strength of nuclear excitation is dependent on them!

Various Operators / Various Hammers/

wooden hammers

metal hammers

hammers
=operators

The sound from the bell is different depending on hammers!

The mode of nuclear excitation is determined by an operator!

***Operators and Excitations

Vibration Modes in Nuclei (Operators)

Microscopic classification of giant resonances

		$\Delta S = 0$, $\Delta T = 0$	$\Delta S = 0$, $\Delta T = 1$	$\Delta S = 1$, $\Delta T = 0$	$\Delta S = 1$, $\Delta T = 1$
L = 0			$\sum \tau_i$ IAS		$\sum \vec{\sigma}_i \tau_i$ GTR
	2nd order	$\sum r_i^2$ ISGMR	$\sum r_i^2 \tau_i$ IVGMR	$\sum r_i^2 \vec{\sigma}_i$ ISSMR	$\sum r_i^2 \vec{\sigma}_i \tau_i$ IVSMR
L = 1			$\sum r_i Y_m^1 \tau_i$ IVGDR	$\sum r_i Y_m^1 \vec{\sigma}_i$ ISSDR	$\sum r_i Y_m^1 \vec{\sigma}_i \tau_i$ IVSDR
	2nd order	$\sum r_i^3 Y_m^1$ ISGDR			
L = 2		$\sum r_i^2 Y_m^2$ ISGQR	$\sum r_i^2 Y_m^2 \tau_i$ IVGQR	$\sum r_i^2 Y_m^2 \vec{\sigma}_i$ ISSQR	$\sum r_i^2 Y_m^2 \vec{\sigma}_i \tau_i$ IVSQR
L = 3		$\sum r_i^3 Y_m^3$ ISGOR	$\sum r_i^3 Y_m^3 \tau_i$ IVGOR	$\sum r_i^3 Y_m^3 \vec{\sigma}_i$ ISSOR	$\sum r_i^3 Y_m^3 \vec{\sigma}_i \tau_i$ IVSOR

Vibration Modes in Nuclei (Operators)

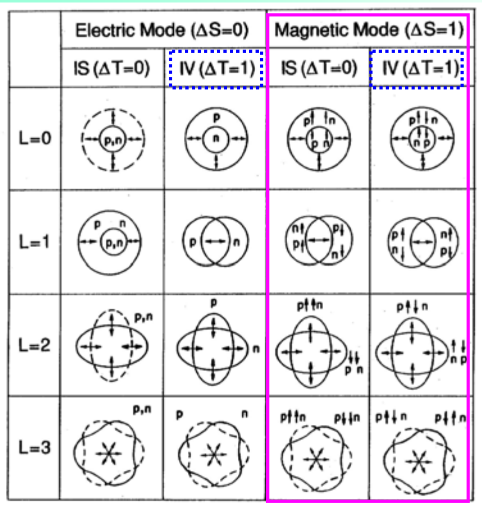
Microscopic classification of giant resonances

		$\Delta S = 0$, $\Delta T = 0$	$\Delta S = 0$, $\Delta T = 1$	$\Delta S = 1$, $\Delta T = 0$	$\Delta S = 1$, $\Delta T = 1$
L = 0			$\sum \tau_i$ IAS		$\sum \vec{\sigma}_i \tau_i$ GTR
	2nd order	$\sum r_i^2$ ISGMR	$\sum r_i^2 \tau_i$ IVGMR	$\sum r_i^2 \vec{\sigma}_i$ ISSMR	$\sum r_i^2 \vec{\sigma}_i \tau_i$ IVSMR
L = 1			$\sum r_i Y_m^1 \tau_i$ IVGDR	$\sum r_i Y_m^1 \vec{\sigma}_i$ ISSDR	$\sum r_i Y_m^1 \vec{\sigma}_i \tau_i$ IVSDR
	2nd order	$\sum r_i^3 Y_m^1$ ISGDR			
L = 2		$\sum r_i^2 Y_m^2$ ISGQR	$\sum r_i^2 Y_m^2 \tau_i$ IVGQR	$\sum r_i^2 Y_m^2 \vec{\sigma}_i$ ISSQR	$\sum r_i^2 Y_m^2 \vec{\sigma}_i \tau_i$ IVSQR
L = 3		$\sum r_i^3 Y_m^3$ ISGOR	$\sum r_i^3 Y_m^3 \tau_i$ IVGOR	$\sum r_i^3 Y_m^3 \vec{\sigma}_i$ ISSOR	$\sum r_i^3 Y_m^3 \vec{\sigma}_i \tau_i$ IVSOR

ΔS=1:
spin excitation

ΔT=1:
IV excitation
(isospin related!)

Vibration Modes in Nuclei (Schematic)



Giant Resonances
(collective excitations)

- absorbs a large fraction of
the total sum rule strength –

See the "Proceedings of Science ENAS 6"
Sep. 18-27, 2011.

Gamow-Teller Giant Resonances for A>90 Nuclei

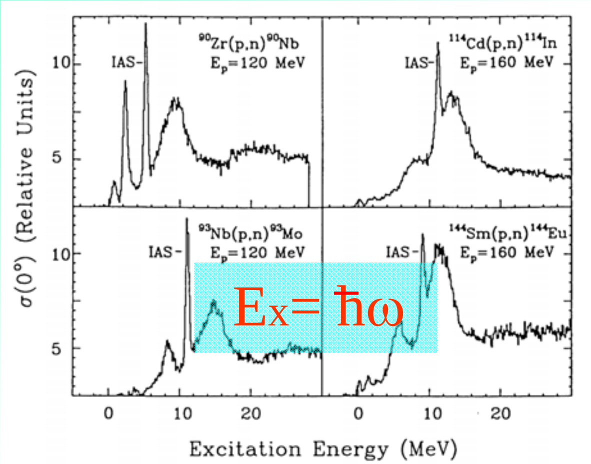
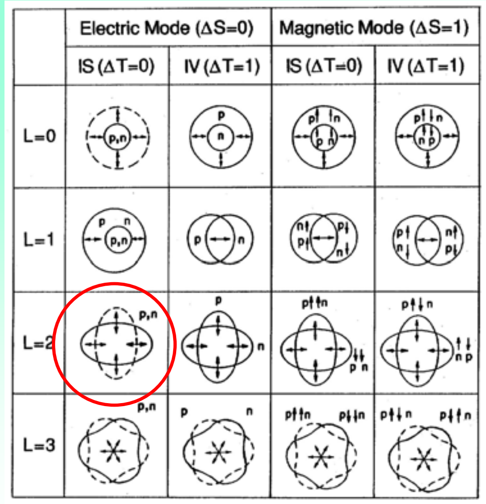


Figure 10 Zero-degree (p,n) spectra for medium A-mass nuclei at the indicated incident energies.

Vibration Modes in Nuclei (Schematic)

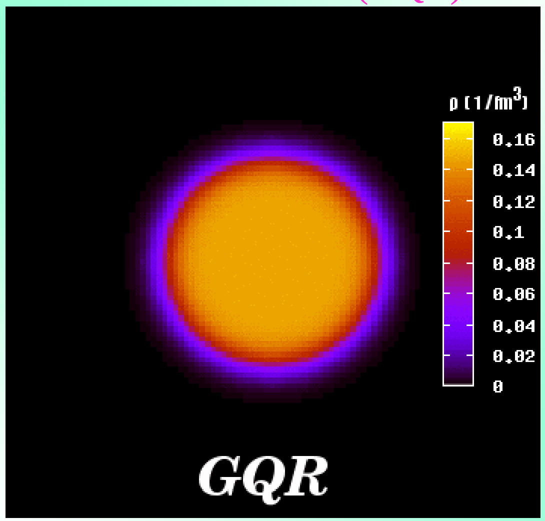


IS-Giant Quadrupole Resonance (GQR)

**IS Electric Giant Resonances

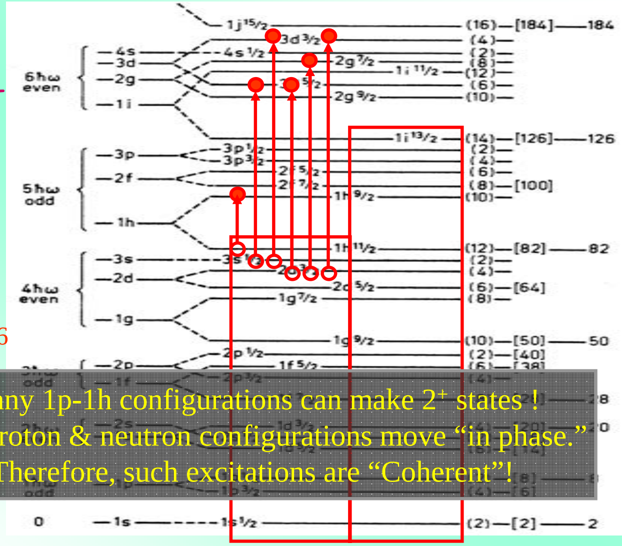
Giant Resonance (GQR)

by M. Itoh



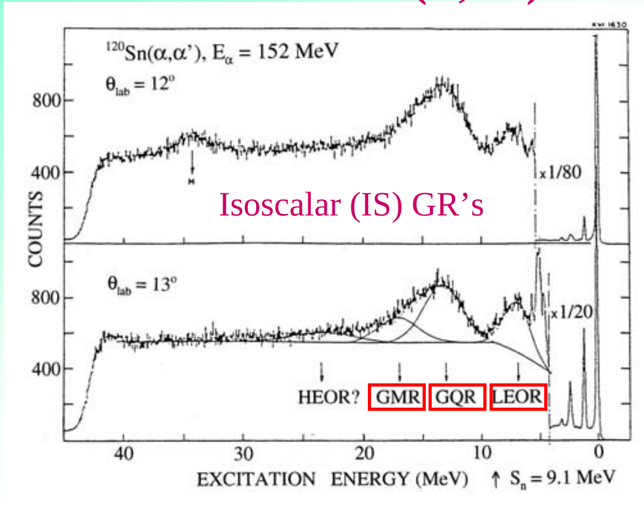
1p-1h Configurations making GQR in ^{208}Pb

Z=82,N=126

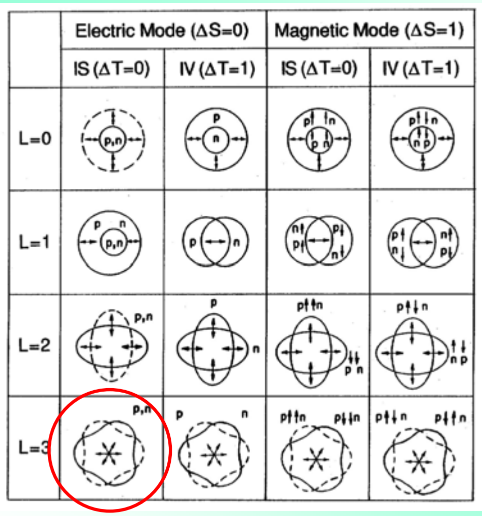


Many 1p-1h configurations can make 2^+ states !
Both proton & neutron configurations move "in phase."
Therefore, such excitations are "Coherent"!

GRs observed in (α, α')

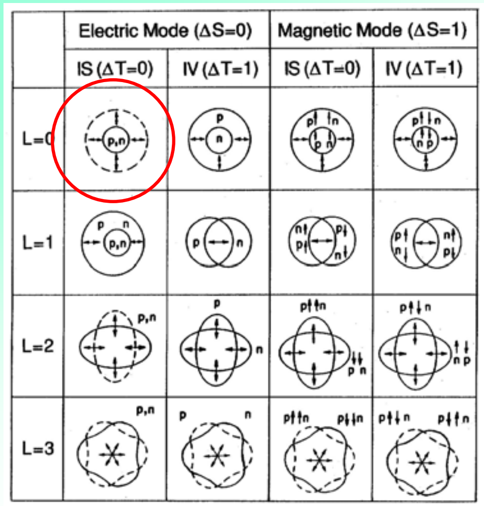


Vibration Modes in Nuclei (Schematic)



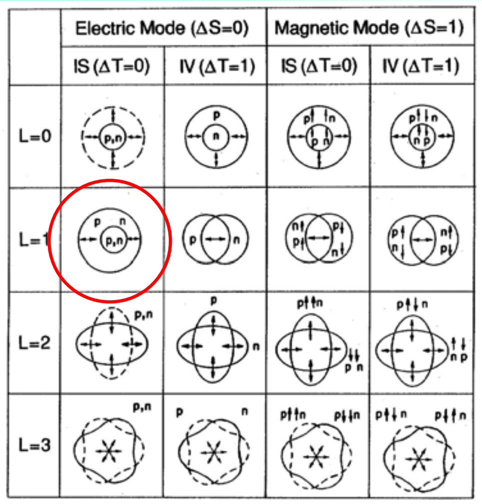
IS-Giant Octupole Resonance (GOR)

Vibration Modes in Nuclei (Schematic)



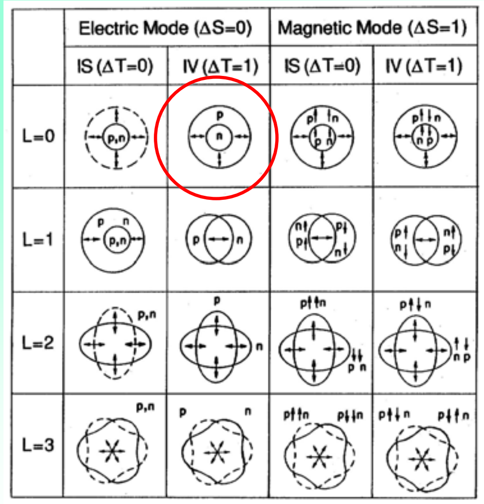
IS-Giant Monopole Resonance (GMR)

Vibration Modes in Nuclei (Schematic)



IS-Giant Dipole Resonance (GDR)

Vibration Modes in Nuclei (Schematic)



IV-Giant Monopole Resonance (IVGMR)

Compression Modes of Nuclei

Macroscopic Picture/Hydrodynamic models/Giant Resonances

Coherent vibrations of nucleonic fluids in a nucleus.

Compression modes : ISGMR, ISGDR

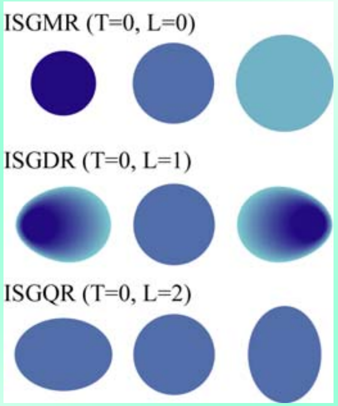
$$E_{ISGMR} = \hbar\sqrt{\frac{K_A}{m\langle r^2\rangle}}$$

$$E_{ISGDR} = \hbar\sqrt{\frac{7}{3}\frac{K_A + \frac{27}{25}\varepsilon_F}{m\langle r^2\rangle}}$$

The nucleus incompressibility:

$$K_A = \left[r^2(d^2(E/A)/dr^2)\right]_{r=R_0}$$

by M. Harakeh



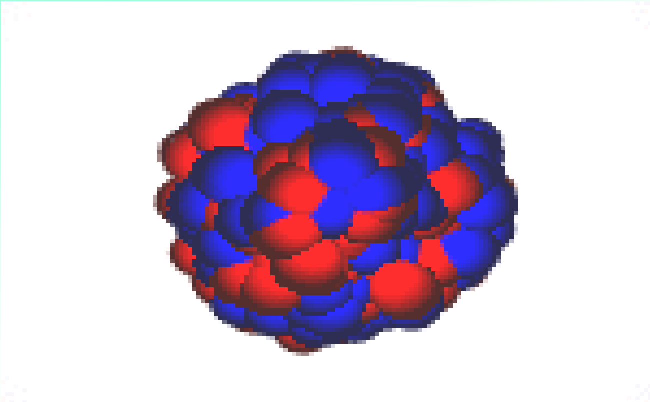
Vibration Modes in Nuclei (Operators)

Microscopic classification of giant resonances

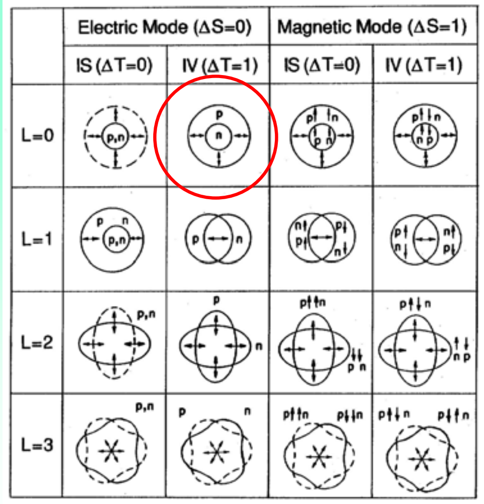
	ΔS = 0, ΔT = 0	ΔS = 0, ΔT = 1	ΔS = 1, ΔT = 0	ΔS = 1, ΔT = 1
L = 0		$\sum \tau_i$ IAS		$\sum \vec{\sigma}_i \tau_i$ GTR
2nd order	$\sum r_i^2$ ISGMR	$\sum r_i^2 \tau_i$ IVGMR	$\sum r_i^2 \vec{\sigma}_i$ ISSMR	$\sum r_i^2 \vec{\sigma}_i \tau_i$ IVSMR
L = 1		$\sum r_i Y_m^1 \tau_i$ IVGDR	$\sum r_i Y_m^1 \vec{\sigma}_i$ ISSDR	$\sum r_i Y_m^1 \vec{\sigma}_i \tau_i$ IVSDR
2nd order	$\sum r_i^3 Y_m^1$ ISGDR			
L = 2	$\sum r_i^2 Y_m^2$ ISGQR	$\sum r_i^2 Y_m^2 \tau_i$ IVGQR	$\sum r_i^2 Y_m^2 \vec{\sigma}_i$ ISSQR	$\sum r_i^2 Y_m^2 \vec{\sigma}_i \tau_i$ IVSQR
L = 3	$\sum r_i^3 Y_m^3$ ISGOR	$\sum r_i^3 Y_m^3 \tau_i$ IVGOR	$\sum r_i^3 Y_m^3 \vec{\sigma}_i$ ISSOR	$\sum r_i^3 Y_m^3 \vec{\sigma}_i \tau_i$ IVSOR

IV Giant Monopole Resonance (IVGMR)

by P. Adrich



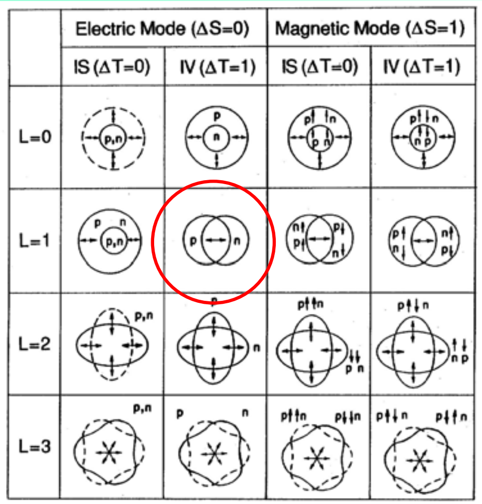
Vibration Modes in Nuclei (Schematic)



Fermi mode

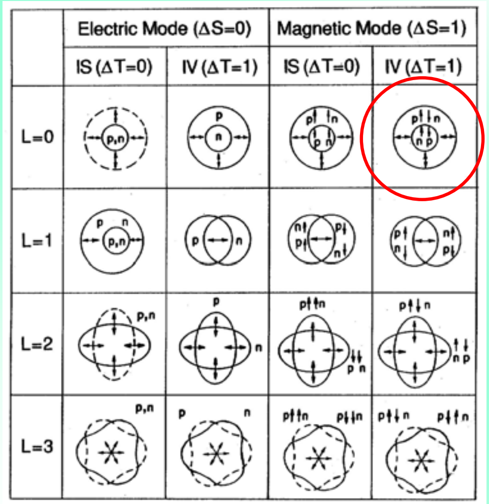
(τ)

Vibration Modes in Nuclei (Schematic)



IV-Giant Dipole Resonance (IVGDR)

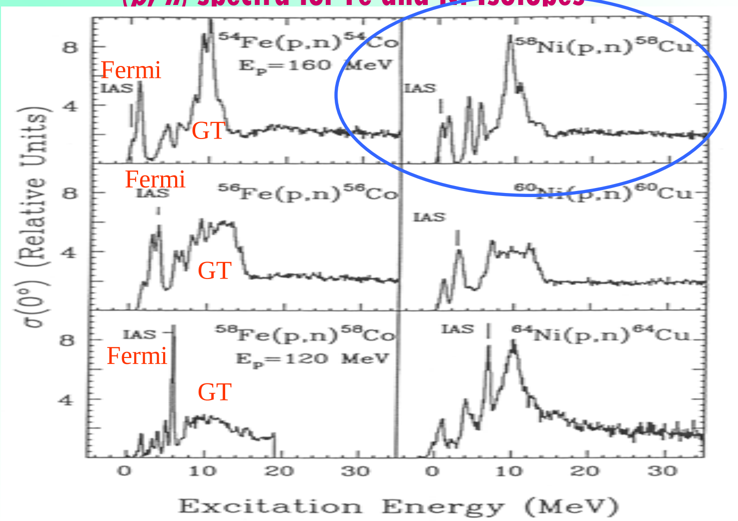
Vibration Modes in Nuclei (Schematic)



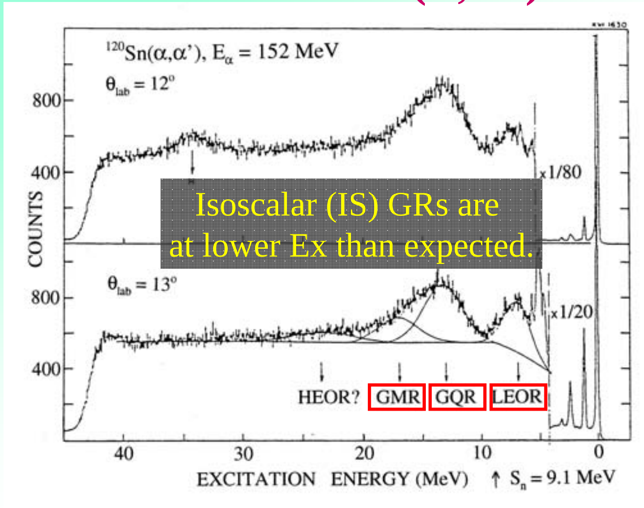
Gamow-Teller mode

($\sigma\tau$)

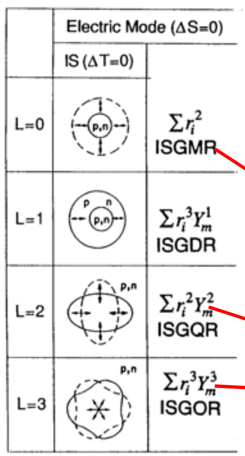
(p, n) spectra for Fe and Ni Isotopes



GRs observed in (α, α')



Vibration Modes and Ex (Harmonic Osc.)

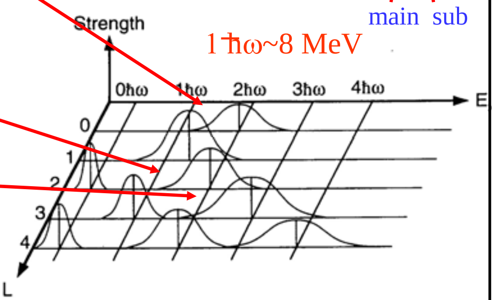


Operator and Excitations

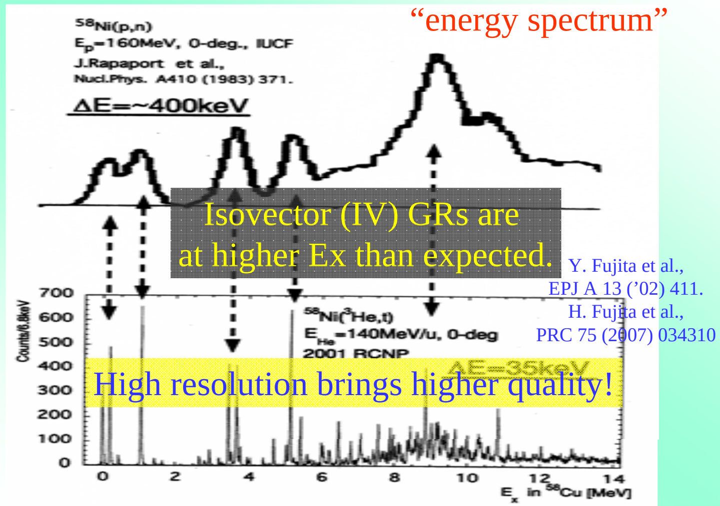
Sherical Harmonic	Radial
Y^1 : ΔL=1	r^1 : Δħω=1
Y^2 : ΔL=2	r^2 : Δħω=2, 0
Y^3 : ΔL=3	r^3 : Δħω=3, 1
:	:

main sub

1ħω~8 MeV



IV excitation: Gamow-Teller



Y. Fujita et al., EPJ A 13 ('02) 411.
H. Fujita et al., PRC 75 (2007) 034310

***Decay and Widths of States

Relationship: Decay and Width

Heisenberg's Uncertainty Principle

$$\Delta x \cdot \Delta p \approx \hbar$$

$$\Delta t \cdot \Delta E \approx \hbar$$

Width $\Gamma = \Delta E$

*if: Decay is Fast,

then: Width of a State is Wider !

*if $\Delta t = 10^{-20}$ sec → ΔE ~100 keV (particle decay

$\Delta t = 10^{-15}$ sec → ΔE ~ 1 eV (fast γ decay)

How are Nuclei defined ?

*Quantum Finite Many-body System of Fermions

=> quantum numbers are important

L, S, J, K, T

=> selection rules of Q-numbers are important

*Active forces in nuclei:

3 out of 4 fundamental forces

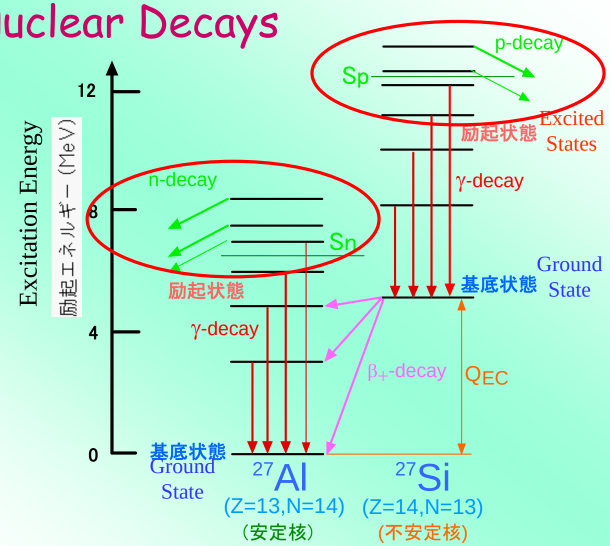
strength: **strong** >> **electro-magnetic** >> **weak**

time :	fast	middle	slow
	(~10^{-20}s)	(~10^{-15}s)	(~10^{-1}s)

*they struggle to make their territory larger !

→phenomena from 3 forces can be combined for the study of nuclei !

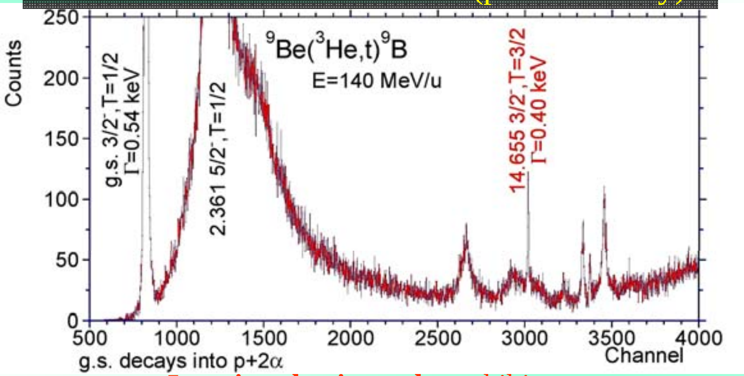
Nuclear Decays



^{9}Be(^{3}He,t)^{9}B spectrum (II)

Width $\Gamma = \Delta E$

*if $\Delta t = 10^{-18}$ sec → ΔE ~1 keV (particle decay)



Isospin selection rule prohibits **proton decay** of **T=3/2** state!

Fermi & Gamow-Teller operators

Fermi operator: τ

ΔL=0, ΔS=0 → ΔJ=0, and ΔT=1 (ΔT=0)

*transition is between the same configuration

Sum rule value: ΣB(F) = $|N\text{-}Z|$

GT operator: $\sigma\tau$

ΔL=0, ΔS=1 → ΔJ=1, and ΔT=1 (ΔT= -1, 0, +1)

*transitions are among *LS*-partner ($j_>$ & $j_<$) configurations

Sum rule value: $|\Sigma B(\text{GT-}) - \Sigma B(\text{GT+})| = 3\,|N\text{-}Z|$

*if $j_i=0^+$ → j_f=1+ j_i=3/2 → $j_f=1/2^+, 3/2^+, 5/2^+$

*if T_i=0 → T_f=1 T_i=1/2 → T_f=1/2, 3/2 T_i=1 → T_f=0, 1, 2

**Sum Rule

As an example, sum rule for the Fermi (& Gamow-Teller) transition is discussed.

**Sum Rule (idea)

Nucleus: quantum finite many-body system

⬇

The number of nucleons involved in each mode (degree of freedom): limited

⬇

Vibration of each mode has a max. amplitude.

⬇

For each operator, sum of transition strength is const.

Sum Rule

★ simple sum rule (non-energy weighted sum rule)

$S = \Sigma \boldsymbol{B}$(operator) = const.

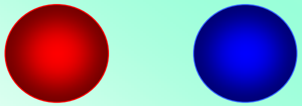
★ energy weighted sum rule

$S = \Sigma E_x \times \boldsymbol{B}$(operator) = const.

Nucleon & Coin

back face

= Coin



proton neutron

= Nucleon

similar mass

nearly the same interaction

T_z=-1/2 T_z= 1/2 isospin T=1/2

Commutation Relationship
(Basic Quantum Mechanics)

*The Uncertainty Principle : $\Delta x \cdot \Delta p_x \sim \hbar/2$

→showing x and p_x are Canonical Operators

(both x and p_x cannot take definite values at the same time)

→thus, commutation relationships

$$\begin{aligned} [x, p_x] &= i\hbar \\ [y, p_y] &= i\hbar \\ [z, p_z] &= i\hbar \end{aligned}$$

are valid !

*Using these relationships, we can obtain commutation relationships for J

$$\begin{aligned} [\mathbf{J}^2, J_\pm] &= 0 \\ [J_+, J_-] &= 2\hbar J_z \\ [J_z, J_\pm] &= \pm\hbar J_\pm \end{aligned}$$

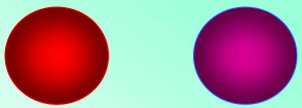
$[T_+, T_-] = 2T_z$

*Note: J and isospin T has the same structure, these relationships are also valid for T

Nucleus & Coin

back front

= Coin



= Nuclei

$^{27}{}_{13}Al_{14}$ $^{27}{}_{14}Si_{13}$ $\boxed{T_z = (1/2)N + (-1/2)Z}$

T_z=1/2 T_z= -1/2 isospin T=1/2, 3/2, …

Sum Rule (example)

*Sum rule value is derived from Commutation Relationship! (very basic!)

ex.1. Fermi transition in β^- decay

$S_{\beta-}(F) = \Sigma_f |\langle f \mid T- \mid i\rangle|^2 = N\text{–}Z$

Think of the value

$\langle i \mid [T_+, T_-] \mid i\rangle = \langle i \mid T_+T_- - T_-T_+ \mid i\rangle$

Left side: $= \langle i \mid 2T_z \mid i\rangle = 2\,(N - Z)/2$

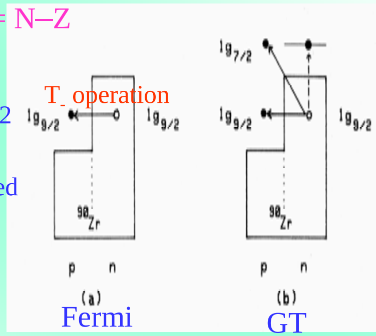
$= N\text{–}Z$

where $[T_+, T_-] = 2T_z$ was used

Right side: using $T_+ \mid i\rangle = 0$

$= \Sigma_f \langle i \mid T_+ \mid f\rangle \langle f \mid T_- \mid i\rangle$

$= \Sigma_f |\langle f \mid T_- \mid i\rangle|^2$



ex.2. Gamow-Teller transition

$\Sigma\beta^-(GT) - \Sigma\beta^+(GT) = 3(N\text{–}Z)$

Sum Rule (example)

*Sum rule value is derived from Commutation Relationship! (very basic!)

ex.1. Fermi transition in β^- decay

$S_{\beta-}(F) = \Sigma_f |\langle f \mid T- \mid i \rangle|^2 = N-Z$

Think of the value

$\langle i \mid [T_+, T_-] \mid i \rangle = \langle i \mid T_+T_- - T_-T_+ \mid i \rangle$

Left side: $= \langle i \mid 2T_z \mid i \rangle = 2\,(N - Z)/2$

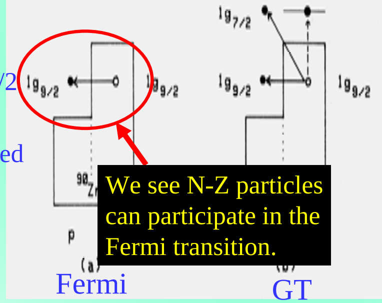
$= N-Z$

where $[T_+, T_-] = 2T_z$ was used

Right side: using $T_+ \mid i \rangle = 0$

$= \Sigma_f \langle i \mid T_+ \mid f \rangle \langle f \mid T_- \mid i \rangle$

$= \Sigma_f |\langle f \mid T_- \mid i \rangle|^2$



ex.2. Gamow-Teller transition

$\Sigma\beta^-(GT) - \Sigma\beta^+(GT) = 3(N-Z)$

Summary

Uniqueness of nuclei : strong, weak, EM int.

What we observe =
reaction mechanism
⊗operator ⊗structure

Operators: IS, IV, Electric, Magnetic

Life time ←→ decay width ←→ interaction strength

Sum Rule: derived from
the Commutation Relationship! (very basic!)