

Statistical and Computational Inverse Problems with Applications

Appendix 2: Examples of informative smoothness priors

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An informative smoothness prior

- Gaussian random variable $\sigma \sim \mathcal{N}(\eta_\sigma, \Gamma_\sigma)$

$$\pi(\sigma) \propto \exp\left(-\frac{1}{2}(\sigma - \eta_\sigma)^T \Gamma_\sigma^{-1} (\sigma - \eta_\sigma)\right) \quad (1)$$

- Write the covariance matrix Γ_σ as

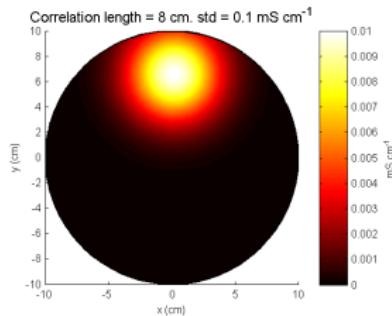
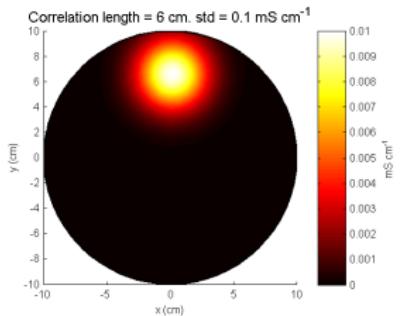
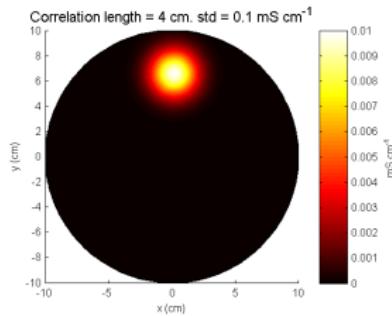
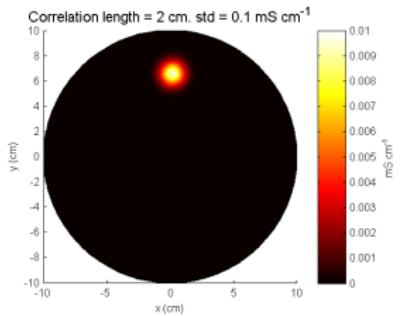
$$\Gamma_\sigma(i, j) = a \exp\left\{-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|_2^2}{2b^2}\right\} \quad (2)$$

where $\mathbf{x}_i \in \mathbb{R}^{2,3}$ is the spatial coordinate corresponding to a discrete conductivity value σ_i (*Lieberman, Willcox, Ghattas 2010*).

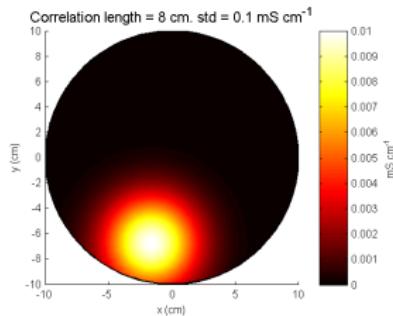
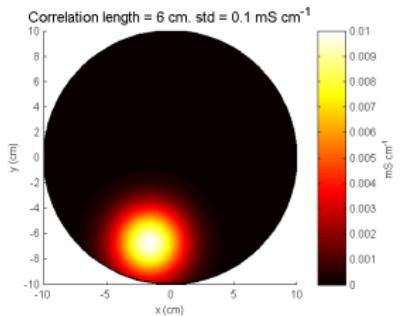
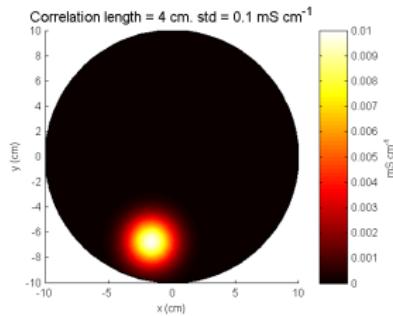
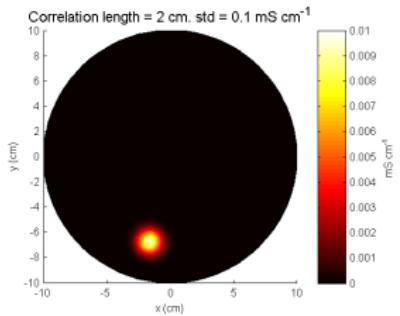
Cross-covariances

Samples

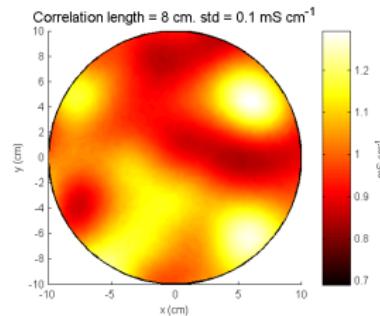
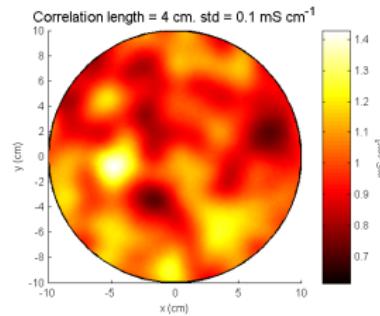
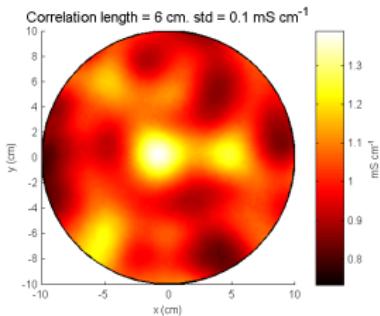
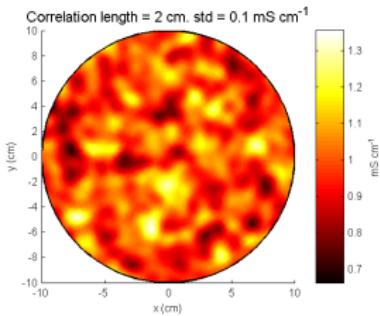
Different correlation lengths: cross-covariances



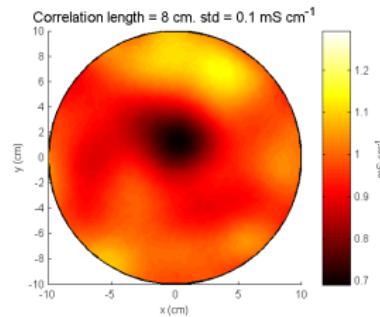
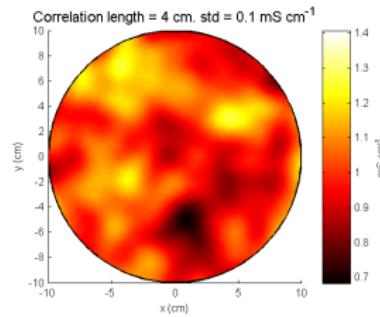
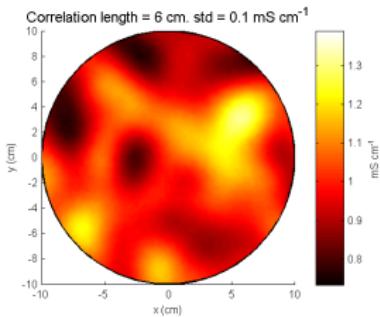
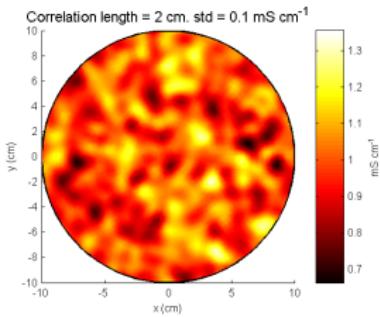
Different correlation lengths: cross-covariances



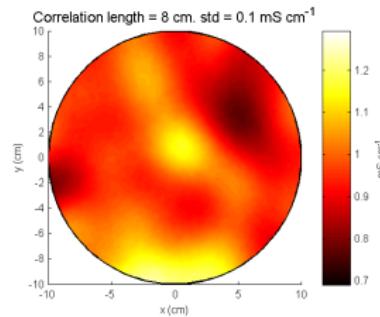
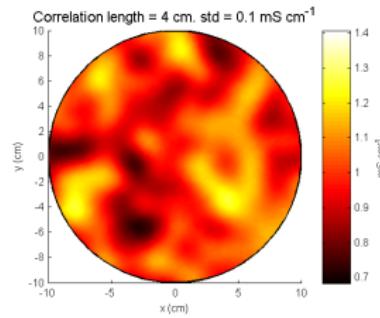
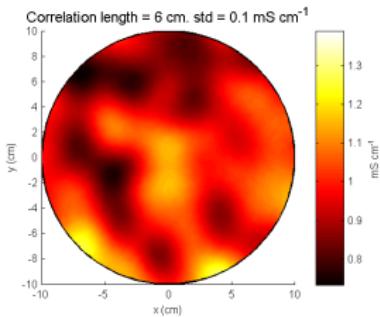
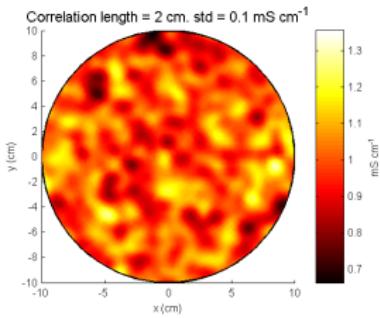
Different correlation lengths: samples



Different correlation lengths: samples



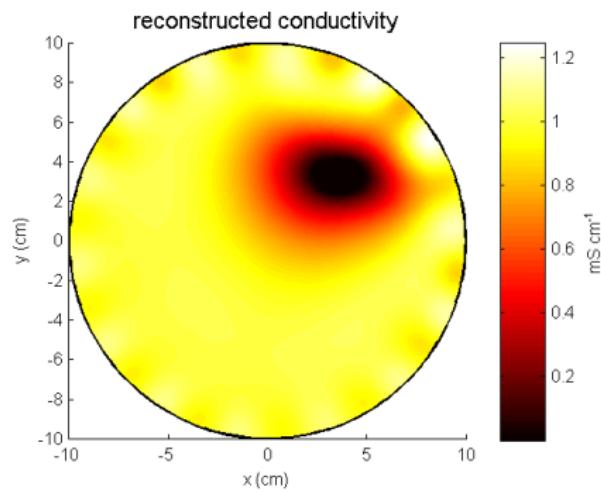
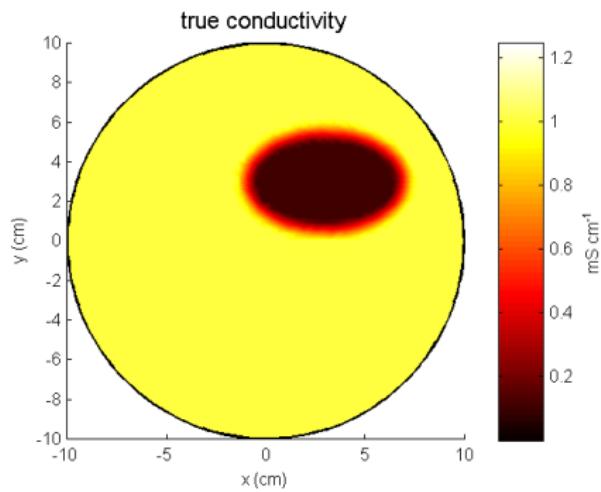
Different correlation lengths: samples



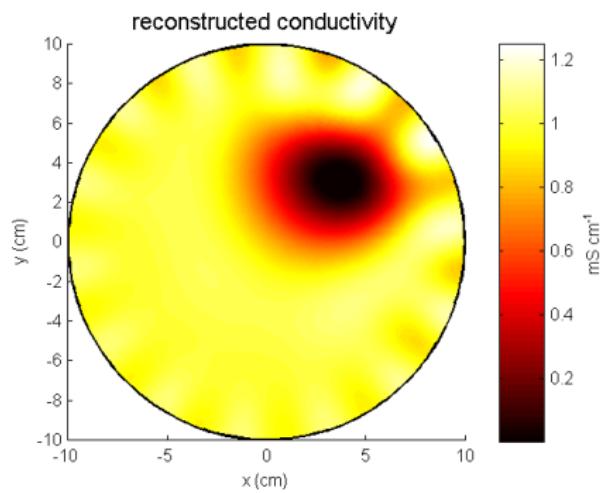
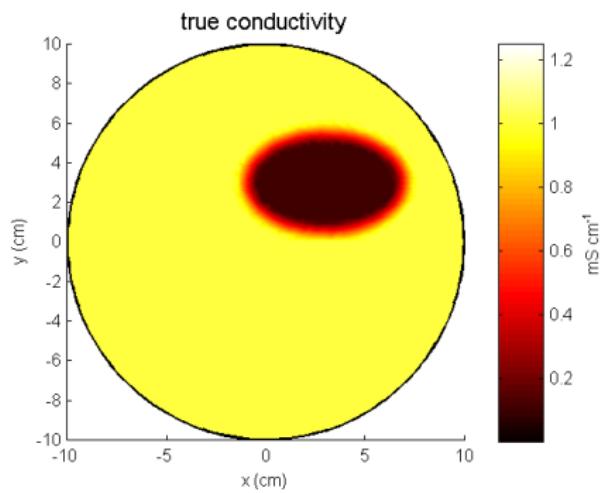
Anisotropic smoothness prior: cross-covariances

Anisotropic smoothness prior: samples

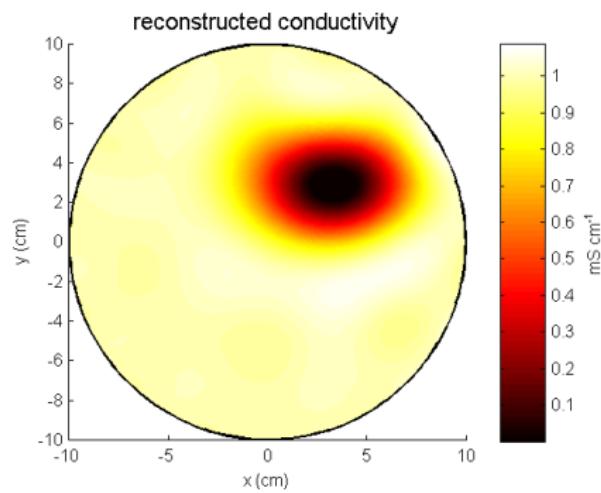
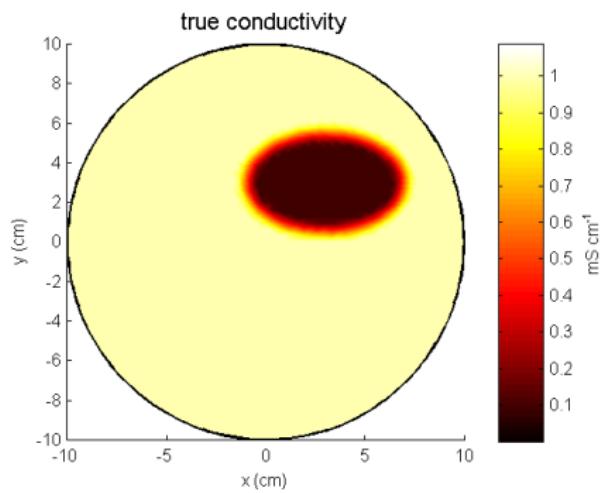
Reconstruction: Isotropic smoothness correlation length 2 cm



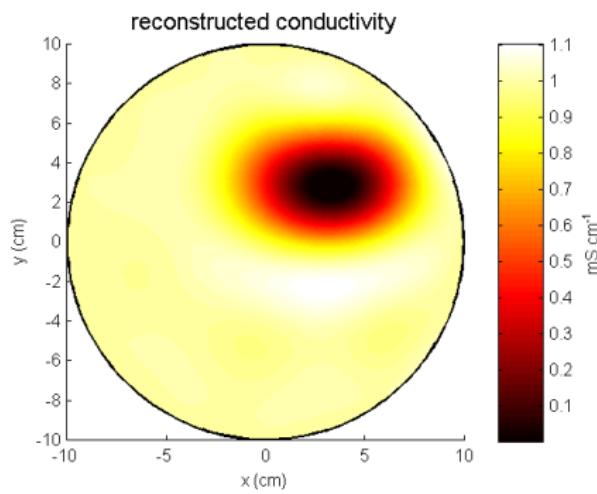
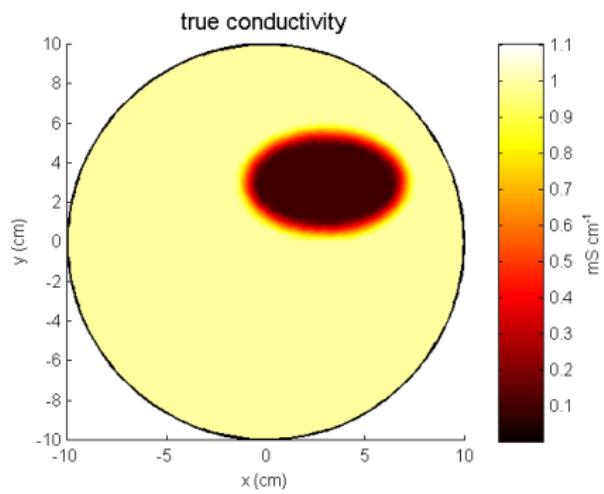
Reconstruction: Isotropic smoothness correlation length 4 cm



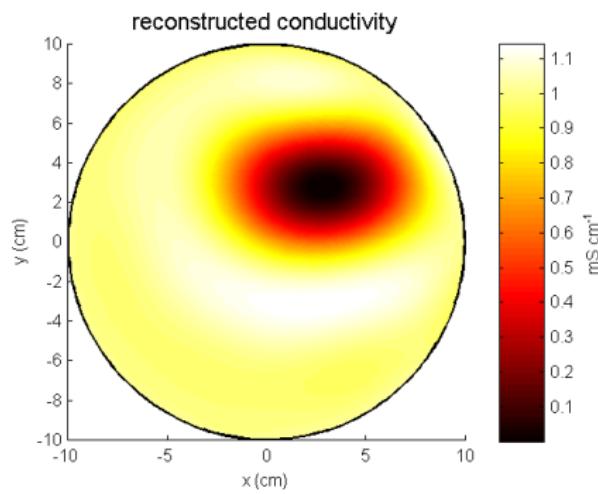
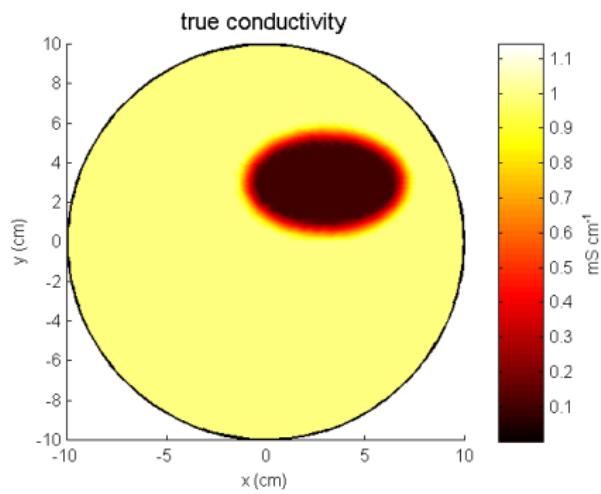
Reconstruction: Isotropic smoothness correlation length 6 cm



Reconstruction: Isotropic smoothness correlation length 8 cm



Reconstruction: Isotropic smoothness correlation length 12 cm



Reconstruction: Anisotropic smoothness correlation lengths; x: 2 cm, y: 8 cm

