University of Jyväskylä
Department of Mathematics and Statistics
MATA280 Foundations of stochastics

Exercise 1
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1.1 *Elementary properties of probability measures.* Prove that any probability measure P on a countable sample space satisfies:

(a) $P(\emptyset) = 0$,
(b) $P(A^c) = 1 - P(A)$,
(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$,
(d) $0 \leq P(A) \leq 1$,
(e) $A \subset B \implies P(A) \leq P(B)$,
(f) $P(B \setminus A) = P(B) - P(A \cap B)$.

1.2 *Probability distributions.* Show that P is a probability distribution on sample space Ω and give an example of a phenomenon that can be modelled by P, when

(a) $\Omega = \{1, 2, 3...\}$, $p \in [0, 1]$ and for all $\omega \in \Omega$

$$P(\omega) = (1-p)^{\omega-1}p. \qquad \text{(Geometric distribution)}$$

(b) $\Omega = \{0, 1, 2, ...\}$, $\lambda > 0$ and for all $\omega \in \Omega$

$$P(\omega) = e^{-\lambda}\frac{\lambda^\omega}{\omega!}. \qquad \text{(Poisson distribution)}$$

(c) $\Omega = \{\omega_1, \omega_2, ..., \omega_N\}$, where $N \in \mathbb{N}$ and for all $\omega \in \Omega$

$$P(\omega) = \frac{1}{|\Omega|}. \qquad \text{(Uniform distribution)}$$

Is it possibel to define uniform distribution on an infinitely numerable space?

1.3 *Product of uniform distributions.* Let the probability function μ_1 be the uniform distribution of the finite set S_1, and let the probability function μ_2 be the uniform distribution of the finite set S_2. Is $\mu_1 \times \mu_2$ then also a uniform distribution? Prove the statement true or provide a counterexample to show it false.

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1.4 *Sum estimate.* Let P be a probability measure on a finite sample space Ω.

(a) Show that for arbitrary events $A_1, \dots, A_k \subset \Omega$ it holds that

$$P\left(\bigcup_{i=1}^{k} A_i\right) \leq \sum_{i=1}^{k} P(A_i). \tag{1}$$

(b) Give an example where the equality holds for the formula (1).

(c) Give an example where the inequality (1) is genuine.

1.5 *Bayes' formula.* Show that if P is a probability measure and $P(A) > 0$, $P(B) > 0$, then

$$P(A \mid B) = \frac{P(A)P(B \mid A)}{P(B)}.$$

1.6 . Quick loans company Strata gives a loan of 150 euros for Pekka and a loan of 200 euros for Antti, both with 50% interest. Let us assume that the borrowers can, with equal probabilities,

(i) pay back the loan with interest,

(ii) pay back the loan, but no interest

(iii) pay neither loan nor interest.

Calculate the profit and loss distribution of Strata. What is the probability that Strata makes profit? How about loss? What further assumption you must make in order to calculate the distribution?