University of Jyväskylä
Department of Mathematics and Statistics
MATA280 Foundations of stochastics

Exercise 2
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2.1 *Passing the course.*

(a) Suppose that five students have same preliminary knowledge in the beginning of the course (and they are independent of each other). Assume that half of the students who do not solve exercise problems pass the course and none of the five students solves problems. What is the probability that all five students pass the course?

(b) Prove the more general result: Let $X_1, ..., X_n$ be independent real-valued random variables on a finite sample space Ω and let $\mathbb{P}$ be a probability measure on Ω. Show that for all $x \in \mathbb{R}$ it holds that

$$\mathbb{P}\big(\min\{X_i,\ i = 1, ..., n\} \geq x\big) = \prod_{i=1}^{n} \mathbb{P}(X_i \geq x).$$

What are X_i's, Ω and $\mathbb{P}$ in the part (a)?

2.2 *Sum and product of random bits.* Let $\theta_1, \ldots, \theta_n$ be independent Bernoulli distributed random variables with parameter $p \in (0, 1)$, so that $\mathbb{P}(\theta_i = 1) = p$ and $\mathbb{P}(\theta_i = 0) = 1 - p$ for all i. Find out the distributions of the following random variablest:

(a) $X = \theta_1 + \theta_2$,

(b) $Y = \theta_1\theta_2$,

(c) $Z = \theta_1 + \cdots + \theta_n$,

(d) $W = \theta_1 \cdots \theta_n$.

2.3 *Independent triplets and pairs.* Let X_1, X_2, X_3 be random integers in a discrete probability space (Ω, P) Are the following statements true or false? Prove them true or show them false by giving a counterexample.

(a) If the random variables X_1, X_2, X_3 are mutually independent, then also the random variables X_i, X_j are mutually independent for all $i \neq j$.

(b) If X_i, X_j are mutually independent for all $i \neq j$, then also X_1, X_2, X_3 are mutually independent.

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2.4 Let $X : \Omega \to S$ and $Y : \Omega \to T$ be random variables. Suppose that $\mathbb{P}(X = s) > 0$ for all $s \in S$. Prove that X and Y are independent if and only if

$$\mathbb{P}(Y = t \,|\, X = s) \;=\; \mathbb{P}(Y = t)$$

for all $s \in S$ and $t \in T$.

2.5 *Conditional probabilities.* Consider problem 6 of Exercise 1. Suppose that the payments from Antti and Pekka are independent and let us denote by random variables X and Y the amounts paid back by Antti and Pekka. Write down examples of events A ja B in terms of X and Y, where

(a) $\mathbb{P}(A \,|\, B) < \mathbb{P}(A)$,

(b) $\mathbb{P}(A \,|\, B) = \mathbb{P}(A)$,

(c) $\mathbb{P}(A \,|\, B) > \mathbb{P}(A)$.