

You should return your solutions as a hard copy or scanned pdf to the instructor of your exercise group by **4pm Mon 29 Nov 2014**.

- 3.1** (a) Suppose that cars arrive at a traffic light junction according to the Poisson distribution. What is the average number of cars that arrive at the junction in one time unit if the parameter is $\lambda > 0$?
- (b) What is the expected value of a random variable that is geometrically distributed?

(Poisson-distribution and geometric distribution were introduced in prob 2 of exercise 1.)

- 3.2** Let X and Y be independent random variables with same distributions, taking values 0 and 1 with equal probability $\frac{1}{2}$. Show that

$$\mathbb{E}((X + Y)|X - Y|) = \mathbb{E}(X + Y)\mathbb{E}(|X - Y|),$$

but $X + Y$ and $|X - Y|$ are not independent.

- 3.3** *Expected values of transforms.* Let X and Y be independent uniformly distributed random numbers in $\{-3, 1, 2\}$. Compute the expectations of the following random numbers:

- (a) $aX + bY$
- (b) XY
- (c) X/Y
- (d) X^Y
- (e) $\sin(\pi/X) \cos(\pi/Y)$

- 3.4** *Expectation as a sum of probabilities.* Let $X : \Omega \rightarrow \mathbb{Z}_+$ is a random variable. Show that

$$\mathbb{E}[X] = \sum_{n=0}^{\infty} \mathbb{P}(X > n).$$

- 3.5** *Quick loans.* Let us consider the problem 6 of exercise 1. Calculate the following
- (a) expected profit of Strata.
 - (b) expected profit of Strata on condition that Antti pays back everything.
 - (c) expected profit of Strata on condition that both pay back something.