University of Jyväskylä
Department of Mathematics and Statistics
MATA280 Foundations of stochastics

Exercise 5
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5.1 *Valtteri's roulette game.* Valtteri plays 300 rounds of roulette by betting one euro at each round to small numbers (1–18). Valtteri's initial capital equals $V_0 = 300$ euros. Denote the value of Valtteri's game account after t rounds byV_t.

(a) Let $U_1, \ldots, U_{300}$ be independent uniformly distributed random variables in $S = \{0, 1, \ldots, 36\}$. Define a function f such that the game account can be represented by,

$$V_t = V_0 + \sum_{s=1}^{t} f(U_s).$$

(b) What is the state space of the random variable V_{300}?

(c) Using part a) compute the expectation $\mathbb{E}V_{300}$.

(d) Let θ_s be the indicator of the event $\{U_s \in [1, 18]\}$. Explain why θ_s follows a $\mathrm{Ber}(p)$ distribution for some p and find out the value of p.

(e) Define a function g such that the game account can be represented as

$$V_t = V_0 + g\left(\sum_{s=1}^{t} \theta_s\right).$$

(f) Using part e) prove that

$$\mathbb{P}\left(\frac{V_{300} - V_0}{V_0} \geq 0.1\right) = \sum_{k=j}^{300} \binom{300}{k} p^k (1-p)^{300-k}$$

for some value of j and find out this value.

(Hint: See problem 2 of Exercise 4.)

5.2 *Binomial distribution.* Let X be a random number following Binomial distribution with parameters $p \in (0, 1)$, $n \geq 2$.

(a) Compute the probability generating function G_X of X.

(b) For which values of t $G_X(t)$ is defined?

(c) Compute the expectation of X using G_X.

(d) Compute the variance of X.

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5.3 *Poisson distribution.* Let X be a Poisson-distributed random number with parameter $\lambda > 0$, so that

$$\mathbb{P}(X = k) \;=\; e^{-\lambda}\frac{\lambda^k}{k!}, \quad k = 0, 1, \ldots$$

(a) Compute the probability generating function G_X of X.

(b) For which values of t $G_X(t)$ is defined?

(c) Compute the expectation of X using G_X.

(d) Compute the variance of X.

5.4 *Sum of Poisson distributed random numbers.* Let X and Y be independent Poisson distributed random numbers with parameters λ_1 and λ_2. Show that $X+Y$ is Poisson distributed.

5.5 *Random sum.* Let $N, X_1, X_2, \ldots$ be independent random numbers taking values in Z_+, where $X_1, X_2, \ldots$ are identically distributed.

(a) Compute the expectation of the random sum $M = \sum_{i=1}^{N} X_i$.

(b) Show that for the variance of the random sum M it holds that

$$\mathrm{Var}(M) = \mathrm{Var}(N)\big(\mathbb{E}[X]\big)^2 + \mathrm{Var}(X)\mathbb{E}[N].$$