An Introduction to Evolutionary Multiobjective Optimization

TIES451

Selected Topics in Soft Computing

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Content

- A brief review of evolutionary optimization.
- Evolutionary multiobjective optimization
  - Basic concepts in multiobjective optimization
- Evolutionary multiobjective optimization (EMO) algorithms
  - Goals
  - Different types of approaches
- Other issues
- Recent developments

A brief review of evolutionary optimization

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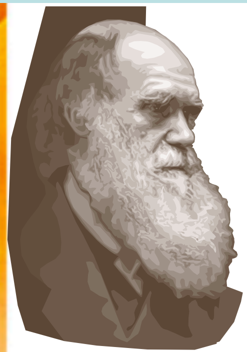
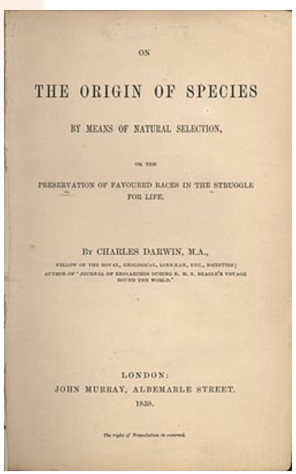
Evolutionary optimization



Charles Darwin



ON

THE ORIGIN OF SPECIES

BY MEANS OF NATURAL SELECTION,

OR THE

PRESERVATION OF FAVOURED RACES IN THE STRUGGLE FOR LIFE.

BY CHARLES DARWIN, M.A.,

LONDON:

JOHN MURRAY, ALBEMARLE STREET.

1859.

Offsprings created by reproduction, mutation, etc.

Natural selection - A guided search procedure

Individuals suited to the environment survive, reproduce and pass their genetic traits to offspring.

Populations adapt to their environment. Variations accumulate over time to generate new species.

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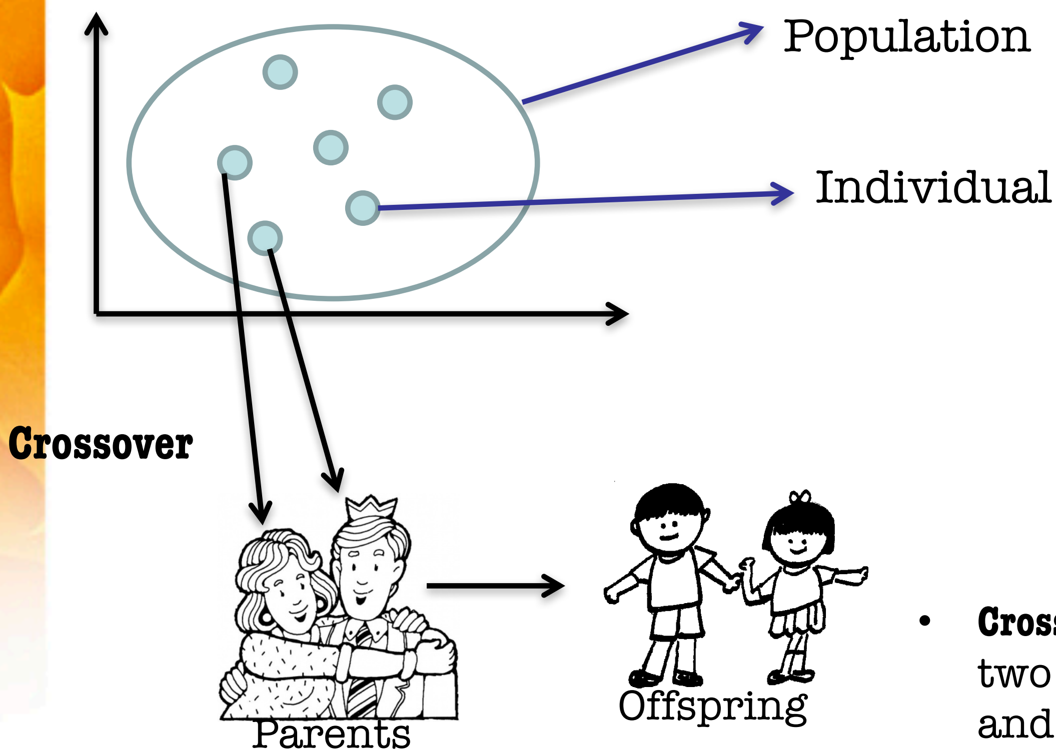


GA and DE which we studied earlier are examples of this kind of algorithm.

- **Crossover**: An operator which involves two or more individuals called parents and creates one or more individual(s) called offspring.
- **Mutation**: An operator which considers only a single individual, perturbs it to create a new offspring.

Mutation

From a single objective to multiple objectives

Transition

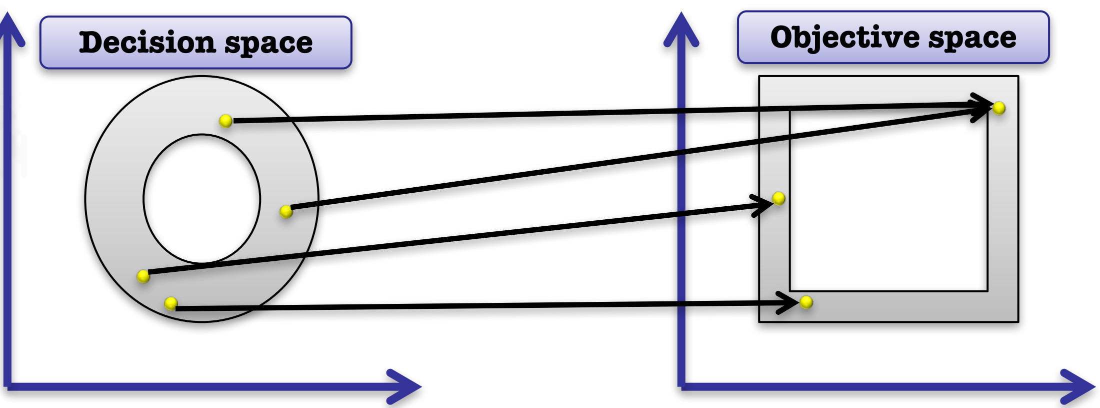
Other objectives can also be considered in addition to the two in the figure.

Multiobjective Optimization

- Let us consider multiobjective optimization problem (MOP) to be of the form:

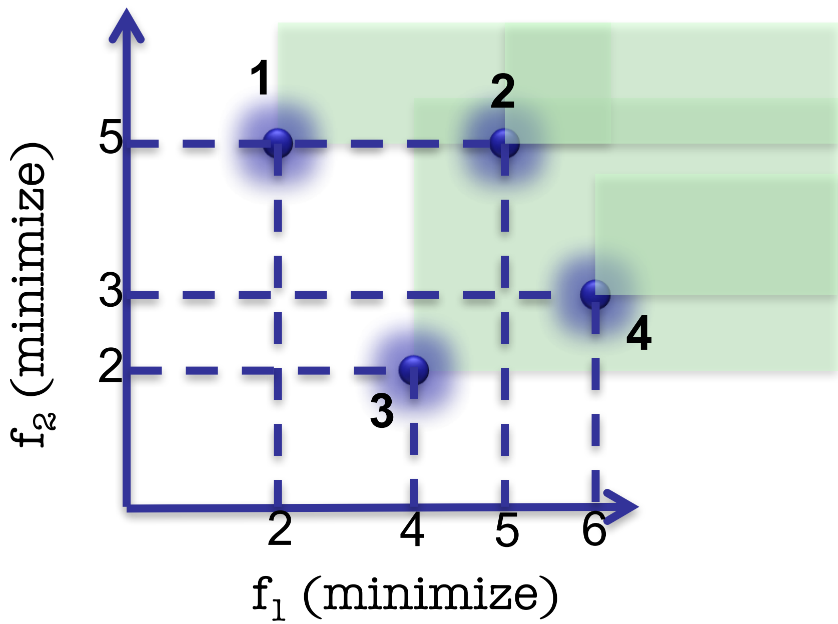
$$\begin{aligned} &minimize \quad \{f_1(x), \ldots, f_k(x)\} \\ &subject\ to \quad x \in S \subset R^n \end{aligned}$$

- with $k \geq 2$ (conflicting) objectives.
- Decision vector /variables: $x = (x_1, \ldots, x_n)$
- Objective vector: $f = (f_1(x), \ldots, f_k(x))$



- Usually a MOP has several optimal solutions with different trade-offs called **Pareto optimal solutions**.
- A decision vector $x^* \in S$ is a Pareto optimal solution for a MOP, if there does not exist another $x \in S$ such that $f_i(x) \leq f_i(x^*)$ for all $i = 1, \ldots, k$ and $f_j(x) < f_j(x^*)$ for at least one index j.
- An objective vector is Pareto optimal, if its corresponding decision vector is Pareto optimal.

- Concept of **non-dominated solutions**:
  - solution **a** dominates solution **b**, if
    - **a** is no worse than **b** in all objectives
    - **a** is strictly better than **b** in at least one objective.



- 3 dominates 2 and 4
- 1 dominates 2

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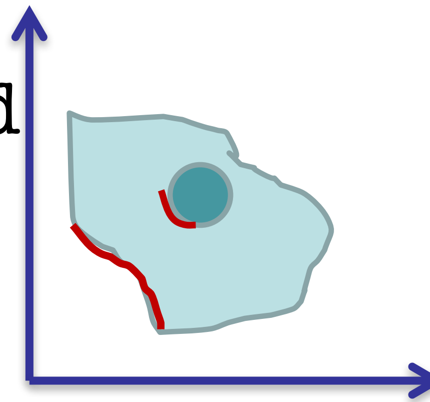
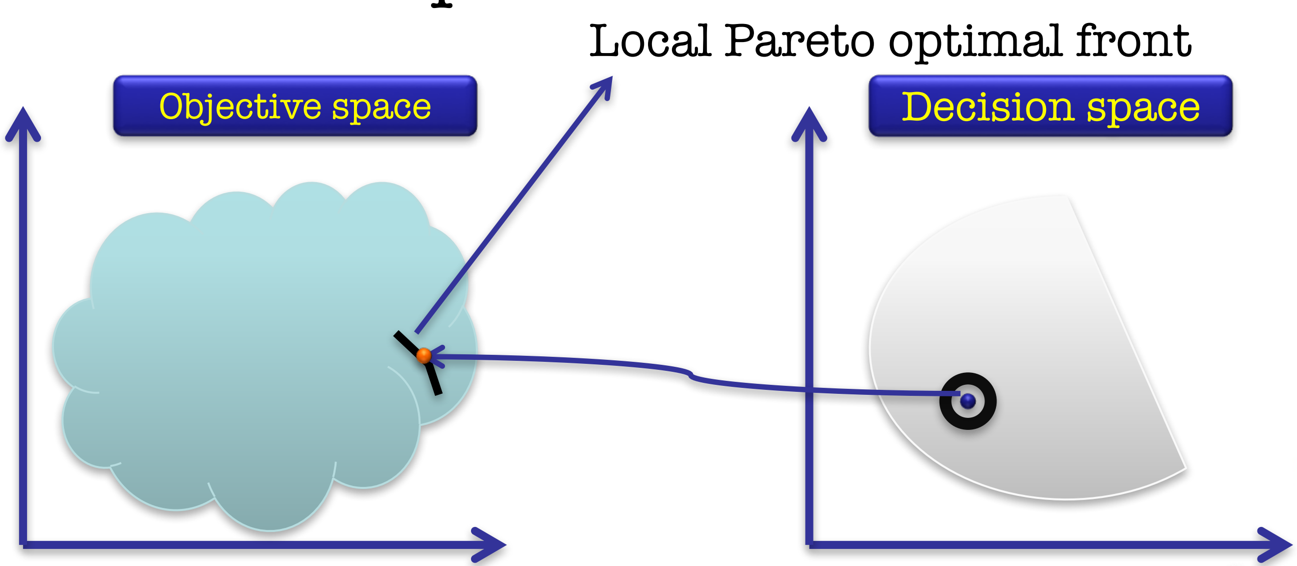
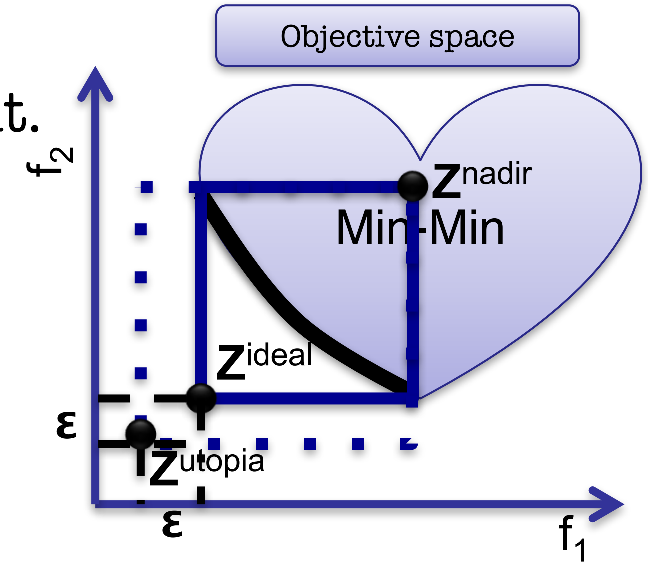
- Pareto-optimal v.s. non-dominated solutions
- Global v.s.

  Local Pareto optimal front





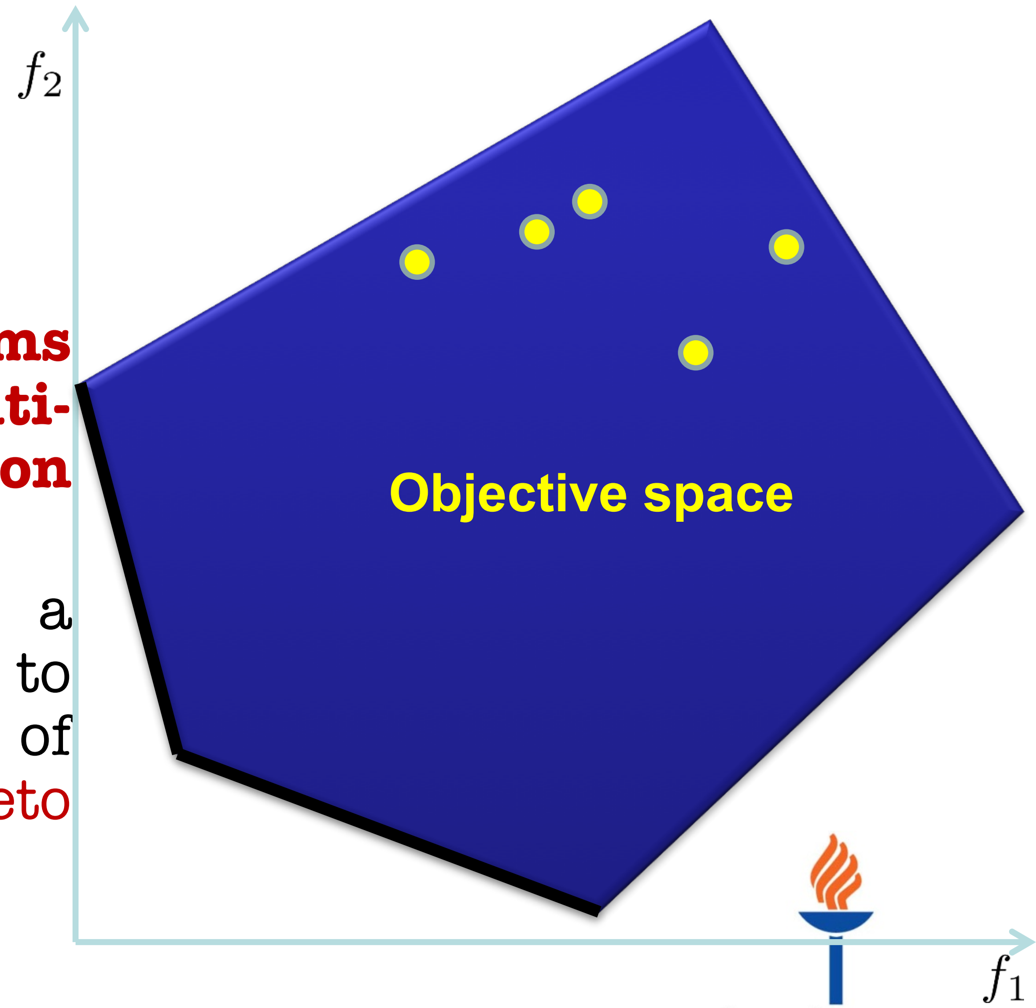
- Ideal point:
  - Usually infeasible
  - lower bound of the Pareto front.
- Nadir point:
  - Upper bound of the Pareto front.
  - Hard to find, but can be approximated
- Normalization of objective vectors:
  - $f^{norm}_i = (f_i - z_i^{utopia})/(z_i^{nadir} - z_i^{utopia})$



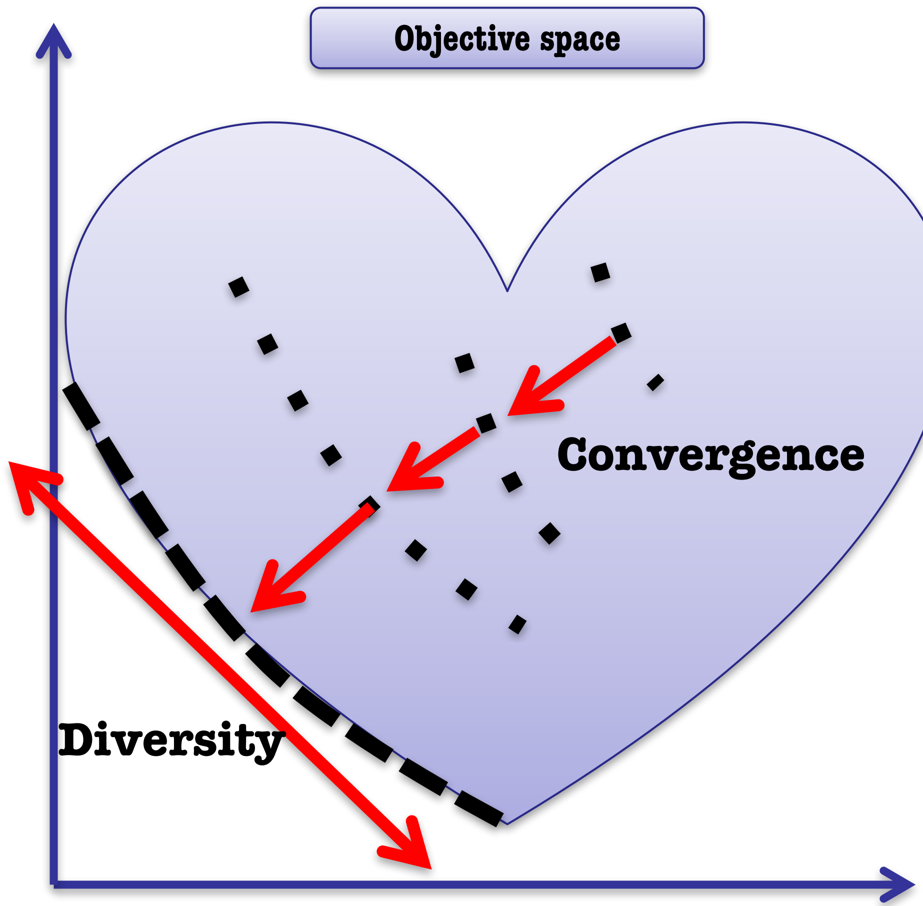
EMO algorithms

EMO algorithms

- What are evolutionary multiobjective optimization algorithms?
  - **Evolutionary algorithms used to solve multi-objective optimization problems**.
- EMO algorithms use a population of solutions to obtain a **diverse set** of solutions close to the Pareto optimal front.



Goals


Objective space
Convergence
Diversity

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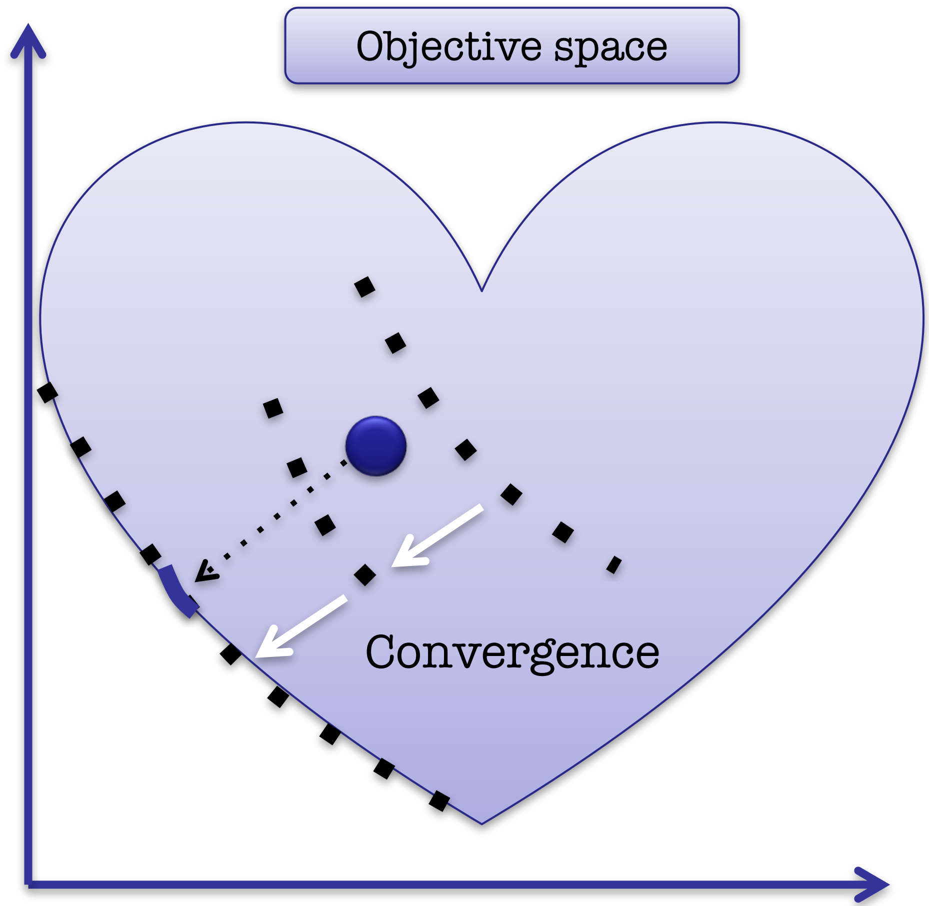
Goals



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- Changes to single objective evolutionary algorithms
  - Fitness computation must be changed
  - Non-dominated solutions are preferred to maintain the drive towards the Pareto optimal front (attain convergence)
  - Emphasis to be given to less crowded or isolated solutions to maintain diversity in the population

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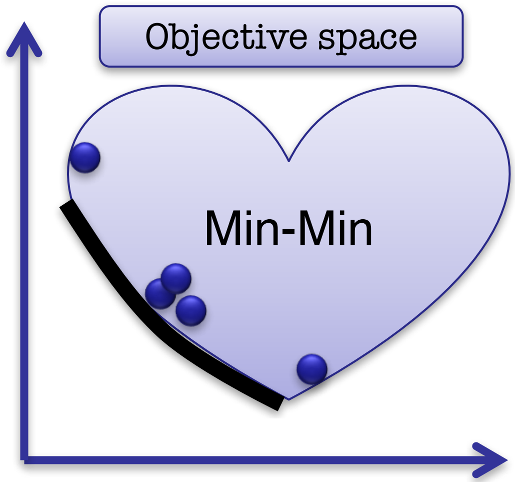
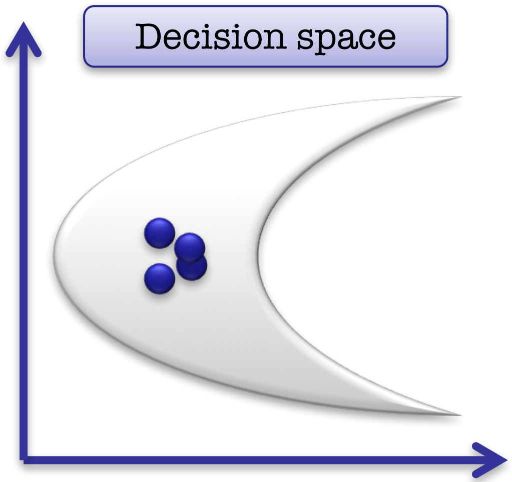
- What are less-crowded solutions ?
  - Crowding can occur in decision space and/or objective phase.
    - Decision space diversity sometimes are needed
      - As in engineering design problems, all solutions would look the same.





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Main Issues in EMO

- How to maintain diversity and obtain a diverse set of Pareto optimal solutions?
- How to maintain non-dominated solutions?
- How to maintain the push towards the Pareto front ? (Achieve convergence)

Algorithm design issues

- The approximation of the Pareto front is itself multi-objective.
  - Convergence: Compute solutions as close as possible to Pareto front quickly.
  - Diversity: Maximize the diversity of the Pareto solutions.
- It is impossible to describe
  - What a good approximation can be for a Pareto optimal front.
  - Proximity to the Pareto optimal front.

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Fitness assignment

- Unlike single objective, multiple objectives exists.
  - Fitness assignment and selection go hand in hand.
- Fitness assignment can be classified in to following categories:
  - Decoposition based
  - Objective based
  - Dominance based

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Decomposition based

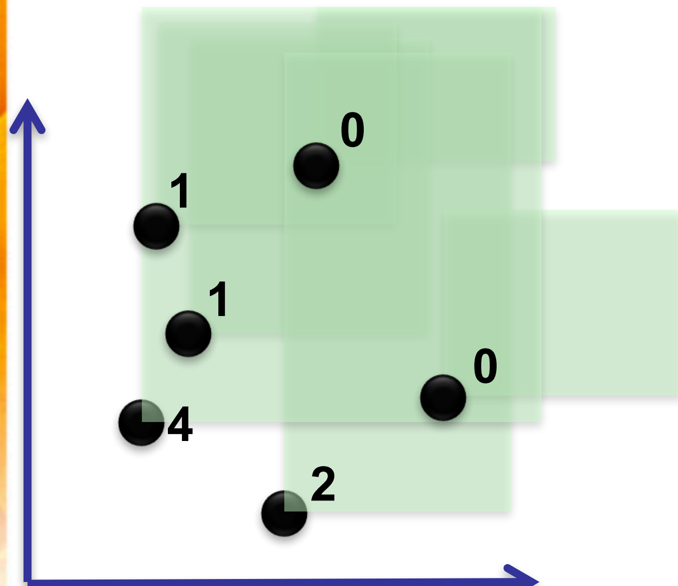
- Decomposes MOPs into a set of scalar subproblems.
- Solves the subproblems simultaneously by evolving a population of solutions
- Any scalarizing technique can be incorporated in to the framework
- e.g., MOEA/D

Objective based

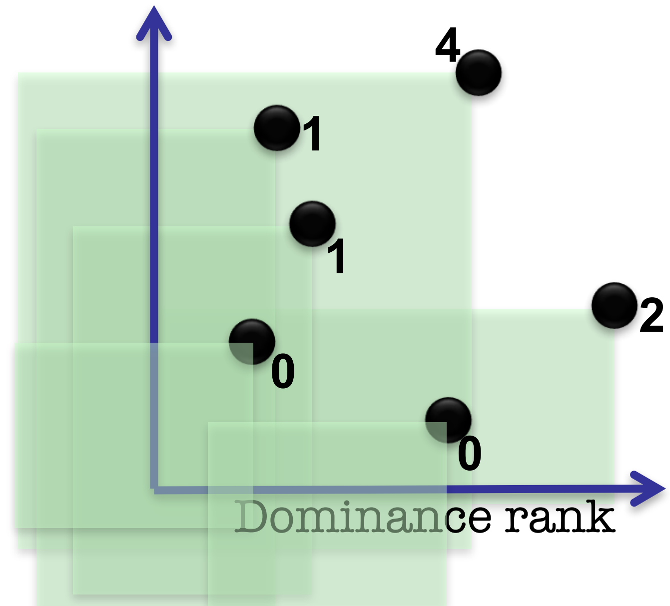
- Switch between objectives in the selection phase.
    - Every time an individual is chosen for reproduction, a different objective decides.
  - E.g. Vector evaluated genetic algorithm (VEGA)

Dominance based

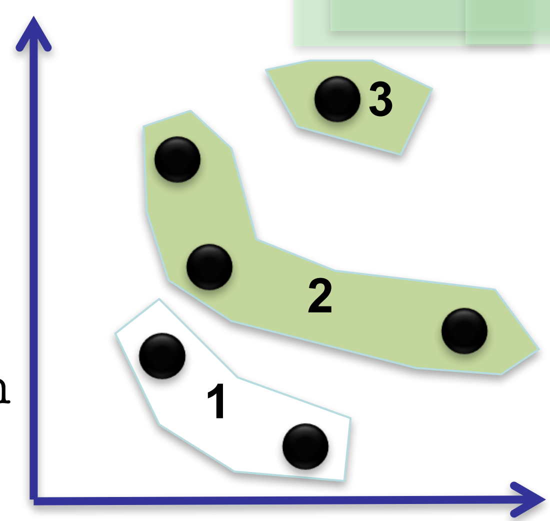
- Pareto dominance based fitness ranking proposed by Goldberg in 1989.
- Different ways
  - **Dominance rank**: Number of individuals by which an individual is dominated.
    - E.g. MOGA, SPEA2
  - **Dominance depth**: The fitness is based on the front an individual belongs.
    - NSGA-II
  - **Dominance count**: Number of individuals dominated by an individual.
    - SPEA2, SIBEA



Dominance count



Dominance rank

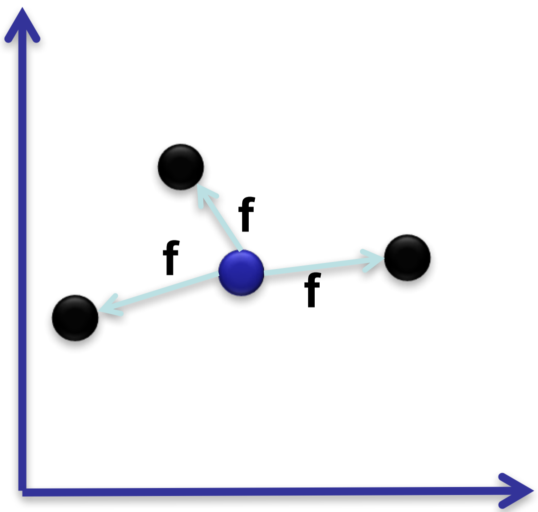


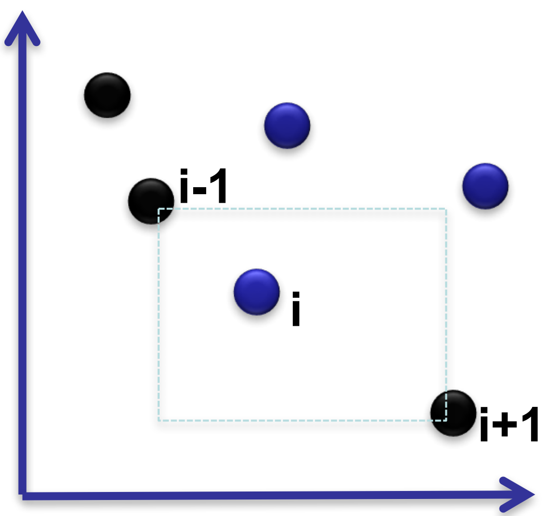
Dominance depth

Diversity preservation

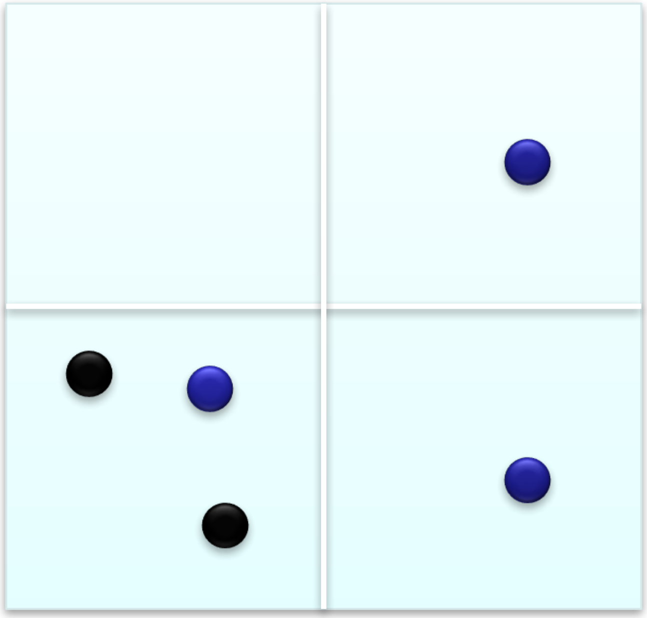
- Chance of an individual being selected
  - **Increases**: Low number of solutions in its neighborhood.
  - **Decreases**: High number of solutions in its neighborhood.
- There are at least three types:
  - Kernel methods
  - Nearest neighbor
  - Histogram

- Kernel methods:
  - Sum of f values, where f is a function of distance.
  - E.g. NSGA
- Nearest neighbor
  - The perimeter of the cuboid formed by the nearest neighbors as the vertices.
  - E.g. NSGA-II





- Histogram
  - Number of elements in a hyperbox.
  - E.g. PAES

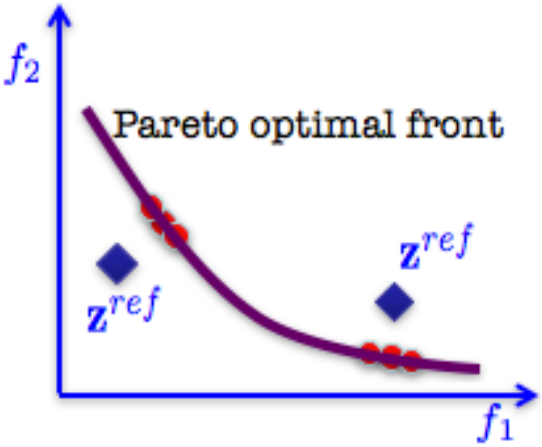


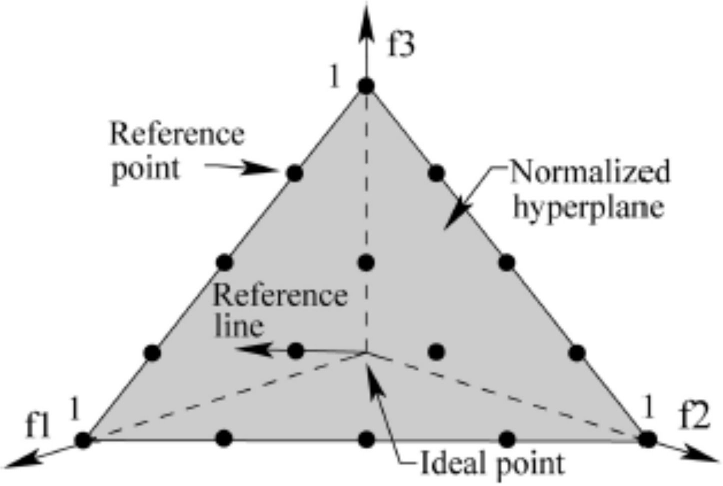
Challenges in EMO algorithms

- Traditional EMO algorithms e.g. NSGA-II cannot handle problems with more than 3 objectives.
  - Need for solutions which increases exponentially with the number of objectives to represent the Pareto optimal front.
  - Lack enough selection pressure towards the Pareto optimal front.
    - Non-dominated solutions increases in the population.
  - Visualization of four and higher dimensional Pareto optimal front is difficult.

Some ways the tackle the challenges

- Instead of entire Pareto optimal front, generate only a smaller subset of preferred Pareto optimal solutions, e.g., R-NSGA-II, I-SIBEA
- Use a pre-defined multiple targeted search: NSGA-III, MOEA/D, RVEA.





Deb & Jain (2014)

Other issues

- Test problems
- Performance assessments

Test problems

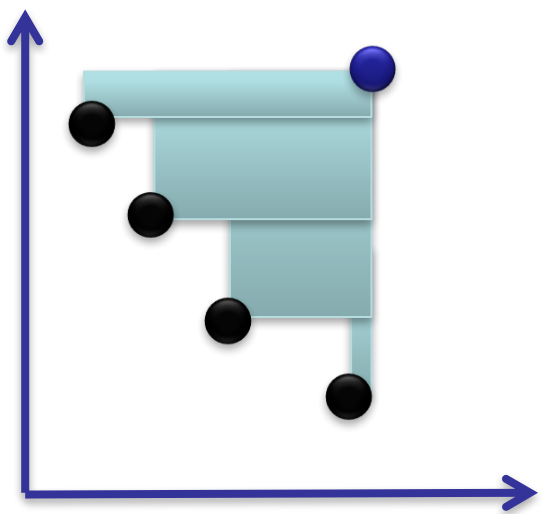
- It is common in the MOEA community to use test problems to test the efficacy of the algorithms proposed.
- However, any MOEA which performs well on the test suite cannot be guaranteed to work well with real world problems.

Performance assessment

- Compare different algorithms to establish efficacy on a set of test problems.
- Measure convergence and diversity of the approximation.
  - Diversity metrics: Spread, spacing etc.
  - Convergence metrics: error ratio, inverted generational distance.
  - Convergence and diversity: Hypervolume, coverage etc.

Performance assessment

- **Hypervolume**: The volume of the space dominated by the approximation in the objective space.



Source codes available:

1. Weighted hypervolume:
   1. http://www.tik.ee.ethz.ch/sop/download/supplementary/weightedHypervolume/
2. Matlab
   1. http://www.mathworks.com/matlabcentral/fileexchange/19651-hypervolume-indicator

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Performance assessment

- Inverted Generational Distance (IGD):

$$IGD(A, P^*) = \frac{\sum_{v \in P^*} d(v, A)}{|P^*|}$$

  - $d(v, A)$ is the minimum Euclidian distance between *v* and the individuals in A.
  - A is the approximation obtained from the MOEA.
  - P^* is the reference set of large enough individuals in the Pareto optimal front.

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- Further topics which will be covered in the course:
  - Constraint handling techniques (covered in the reading assignment)
  - Hybrid EMO / memetic algorithms and genetic programming (covered in a discussion session)

Recent developments