Table 1: Objective function values of individuals

Individual	$f(x)$
1	$(0, 1)$
2	$(1, 0)$
3	$(2, 1.5)$
4	$(1.5, 3)$
5	$(3, 1.6)$
6	$(4, 3.5)$
7	$(4.5, 3.1)$
8	$(5, 2.5)$
9	$(6, 5)$
10	$(5.5, 7)$
11	$(4.2, 6)$
12	$(3.3, 6.5)$

1 Exercises related to NSGA-II

NSGA-II introduces a fast non-dominated sorting algorithm and a method to assign crowding distance to each solution (individual). Now suppose that you have a population consisting of 12 individuals in a bi-objective optimization problem. The objective function values of the 12 individuals are listed in Table 1. Please do the following:

- Implement the fast non-dominated sorting algorithm and sort the 12 individuals into different fronts.
- Calculate the crowding distance of each individual.
- Select 6 individuals based on the partial order $\prec_n$.

When returning the results of this exercise, please return the following:

- Your implementations of the fast non-dominated sorting and the calculation of the crowding distance.
- Results of the fast non-dominated sorting.
- The crowding distances of each individual.
- The selected 6 individuals.

2 Exercise related to MOEA/D

We consider running MOEA/D on the following problem for two generation with a neighbor size $T = 4$:

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^{2}[-10\exp(-0.2\sqrt[2]{x_i^2 + x_{i+1}^2})] \\ & \sum_{i=1}^{3}[|x_i|^{0.8} + 5\sin(x_i^3)] \\ \text{subject to} & -5 \leq x_i \leq 5 \\ & 1 \leq i \leq 3 \end{array} \tag{1}$$

The ideal objective vector of the problem is $z^* = (-20, -12)$. The initial population is given in Table 2, and we have the uniformly distributed weights as in Table 3.

Please carry out the following exercise:

- Use Tchebycheff approach to evaluate the fitness of the individuals and follow the MOEA/D algorithm to generate the next generation.
- Use weighted sum approach to evaluate the fitness of the individuals and follow the MOEA/D algorithm to generate the next generation.

Individual	x
1	$(-1.0074, -3.0188, 4.8305)$
2	$(0.2688, -0.1031, -1.9855)$
3	$(-0.8320, -1.6051, 2.0110)$
4	$(1.5686, 4.5163, 1.6634)$
5	$(1.2797, 4.2033, 0.3913)$
6	$(-2.0802, -4.4732, 1.9811)$
7	$(-0.6835, 2.3786, 1.6653)$
8	$(-4.8451, -2.3088, -3.2187)$
9	$(4.8406, -0.7716, -3.7199)$
10	$(-3.3283, 0.4787, 4.9908)$
11	$(-3.9378, 4.4274, -3.2888)$
12	$(-1.2759, -0.8226, -4.6740)$

Table 2: Individuals

Individual	λ
1	$(0, 1.0000)$
2	$(0.0909, 0.9091)$
3	$(0.1818, 0.8182)$
4	$(0.2727, 0.7273)$
5	$(0.3636, 0.6364)$
6	$(0.4545, 0.5455)$
7	$(0.5455, 0.4545)$
8	$(0.6364, 0.3636)$
9	$(0.7273, 0.2727)$
10	$(0.8182, 0.1818)$
11	$(0.9091, 0.0909)$
12	$(1.0000, 0)$

Table 3: Weights for each individuals

When submitting the exercise, please provide the following information:

- Your implementation
- The nearest four neighbors of each individual.
- Tchebycheff function values and the weighted sum function values of each individual.
- The next generation (with information on what kind of genetic operators are used). Because of using two different approach to evaluate the fitness of the solutions, you will have two populations in the next generation.

Submit your implementations and your results by 8,15, 8th of March to yue.y.zhou-kangas@jyu.fi with a subject TIES451_exercise4_yoursurname.