Squeezed states of light

D. F. Walls

Physics Department, University of Waikato, Hamilton, New Zealand

The properties of a unique set of quantum states of the electromagnetic field are reviewed. These 'squeezed states' have less uncertainty in one quadrature than a coherent state. Proposed schemes for the generation and detection of squeezed states as well as potential applications are discussed.

THE electric field for a nearly monochromatic plane wave may be decomposed into two quadrature components with time dependence cos ωt and sin ωt respectively. In a coherent state, the closest quantum counterpart to a classical field, the fluctuations in the two quadratures are equal and minimize the uncertainty product given by Heisenberg's uncertainty relation. The quantum fluctuations in a coherent state are equal to the zero-point fluctuations and are randomly distributed in phase. These zero-point fluctuations represent the standard quantum limit to the reduction of noise in a signal. Even an ideal laser operating in a pure coherent state would still possess quantum noise due to zero-point fluctuations.

Other minimum uncertainty states are possible which have less fluctuations in one quadrature phase than a coherent state at the expense of increased fluctuations in the other quadrature phase. Such states, which have been called squeezed states[1–5] (other names include two photon coherent states, generalized coherent states), no longer have their quantum noise randomly distributed in phase. Such states offer intriguing possibilities. In the present optical communication systems which use coherent beams of laser light propagating in optical fibres, the ultimate limit to the noise is given by the quantum noise or zero-point fluctuations. If, instead, beams of squeezed light were used to transmit information in the quadrature phase that had reduced fluctuations the quantum noise level could be reduced below the zero-point fluctuations. Optical communication systems based on light signals with phase sensitive quantum noise have been proposed by Yuen and Shapiro[6,7].

The concept of squeezed states applies to other quantum mechanical systems. For example, they may have a role in increasing the sensitivity of a gravitational wave detector. A standard bar detector for gravitational radiation may be treated as a harmonic oscillator. The effect of the gravitational radiation is so weak that the expected displacement of the bar is of the order of 10^{-19} cm. This is the same order of magnitude as the quantum mechanical uncertainty of the bar's position in its ground state. Thus the signal from the gravitational wave detector may be obscured by the zero-point fluctuations of the detector. This is a striking example of the influence of quantum fluctuations on a macroscopic system. In principle, a way of beating this problem is clear. Instead of the ground state of the oscillator with its quantum noise randomly distributed in phase one prepares the oscillator in a squeezed state. One then measures the displacement due to the gravitational radiation in the quadrature with reduced fluctuations. In this way it should be possible to detect displacements less than the quantum mechanical uncertainty in the bar's position. Of course, this leaves a lot of technical questions unanswered. How does one prepare the bar in a squeezed state? How does one make a measurement on the bar's quadrature phase? These problems and suggested solutions are discussed elsewhere[8,9] in treatments of quantum non-demolition measurements.

The statistical properties of light fields such as coherent or thermal light may be calculated by techniques similar to classical probability theory using an expansion of the density operator in terms of coherent states, the Glauber–Sudarshan P representation[10,11]. Coherent light has poissonian photon counting statistics. Squeezed states of light on the other hand may have sub-poissonian photon counting statistics and have no nonsingular representation in terms of the Glauber–Sudarshan P distribution. The statistical properties of such fields cannot be calculated by techniques analogous to classical probability theory. Squeezed states are, therefore, an example of a nonclassical light field. To be precise we shall define a nonclassical light field as one that has no positive nonsingular Glauber–Sudarshan P function.

Another example of a nonclassical light field is a number state. This certainly has no nonsingular Glauber–Sudarshan P function and clearly has sub-poissonian photon statistics. Such nonclassical light fields with sub-poissonian photon statistics which exhibit photon antibunching have been observed experimentally[12,13,50]. A number state, however, has its quantum fluctuations randomly distributed in phase and hence does not exhibit squeezing. While a squeezed state may exhibit sub-poissonian photon statistics and hence photon antibunching it is not a necessity. Sub-poissonian statistics result if the quadrature phase with reduced fluctuations carries the coherent excitation. Using photon counting techniques direct measurements of the intensity fluctuations of a light field are possible. To determine the fluctuations in the quadrature phases a phase sensitive detection scheme is necessary. This can be achieved by homodyning or heterodyning the signal with a local oscillator followed by photon counting measurements. To generate a squeezed state a phase dependent nonlinear optical process is necessary.

Phase dependent correlation functions

Detection of a light signal with a photon counter yields a measurement of the light intensity $I(t)$ or photon number $n(t)$. Using electronic correlators one may then compute the intensity or photon number correlations of the light field. For example, one may measure the normalized second-order correlation function

$$g^{(2)}(0)=\frac{\langle :I^2: \rangle}{\langle I \rangle^2} \tag{1}$$

where : : denotes normal ordering of the quantum mechanical operators. For sufficiently short counting times the variance $V(n)$ of the photon number distribution is related to $g^{(2)}(0)$ by

$$\frac{V(n)-\langle n \rangle}{\langle n \rangle}=g^{(2)}(0)-1 \tag{2}$$

A coherent light field with poissonian statistics has $g^{(2)}(0)=1$. Thermal light which has increased intensity fluctuations has $g^{(2)}(0)=2$. Since $g^{(2)}(0)$ represents the probability of two photons arriving simultaneously this is referred to as photon

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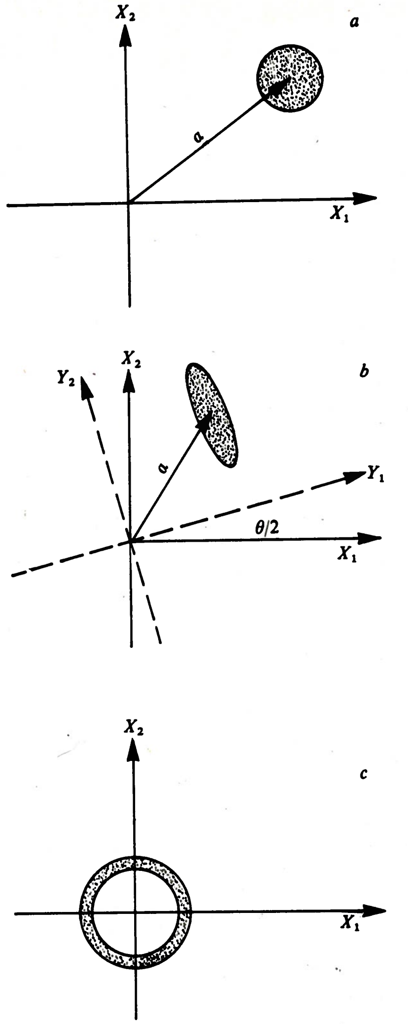

Fig. 1 Phase space plot showing the uncertainty in: *a*, a coherent state $|\alpha\rangle$; *b*, a squeezed state $|\alpha, r\,e^{i\theta}\rangle$ $(r>0)$; *c*, a number state $|n\rangle$.

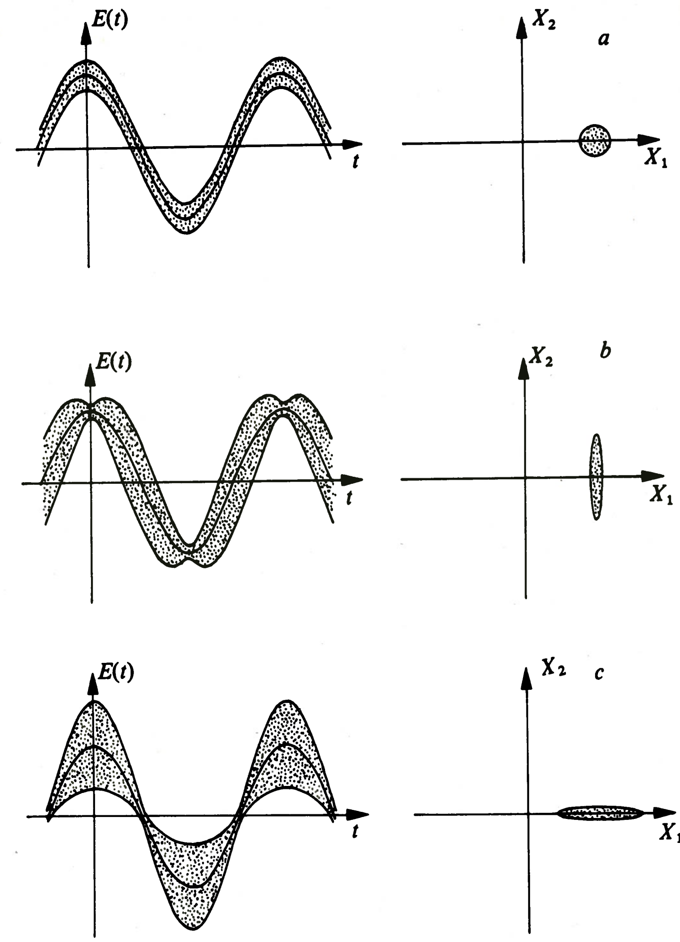

Fig. 2 Plot of electric field against time showing the uncertainty for: *a*, a coherent state $|\alpha\rangle$ (α real); *b*, a squeezed state $|\alpha, r\rangle$ with reduced amplitude fluctuations (α real, $r>0$); *c*, a squeezed state $|\alpha, r\rangle$ with reduced phase fluctuations (α real, $r<0$). Reproduced with permission from Caves[21].

bunching. A light field with sub-poissonian statistics will have $g^{(2)}(0)<1$, an effect known as photon antibunching. Photon antibunching is a quantum mechanical effect which may not be derived from a classical description of the field. Such fields do not have a positive nonsingular representation in terms of the Glauber–Sudarshan P distribution which expresses the density operator for a single mode field as[10,11]

$$\rho = \int P(\alpha)|\alpha\rangle\langle\alpha|\, d^2\alpha \tag{3}$$

where $|\alpha\rangle$ is a coherent state. This representation has found considerable application in optics because the taking of quantum mechanical averages resemble classical averaging procedures provided $P(\alpha)$ exists as a positive nonsingular function. For fields which exhibit photon antibunching, however, the $P(\alpha)$ are highly singular functions. In this sense we say that such fields are nonclassical. The quantum theory of light received further verification when photon antibunching was observed experimentally in resonance fluorescence from a two level atom[12,13] in agreement with theoretical predictions[14–16] (for reviews see refs 17–19).

Our discussion of the properties of phase dependent correlation functions is illustrated with reference to a single mode field. We may write the electric field as

$$E(t) = \lambda(a\, e^{-i\omega t} + a^\dagger e^{i\omega t}) \tag{4}$$

where λ is a constant including the spatial wave functions. In the quantum theory of radiation the amplitudes a and $a^\dagger$ are quantum mechanical operators which obey boson commutation relations. We may write

$$a = X_1 + iX_2 \tag{5}$$

where X_1 and X_2 are hermitian operators obeying the commutation relation

$$[X_1, X_2] = \frac{i}{2} \tag{6}$$

In terms of X_1 and X_2 one may write $E(t)$ as

$$E(t) = \frac{\lambda}{2}(X_1 \cos \omega t + X_2 \sin \omega t) \tag{7}$$

Thus X_1 and X_2 may be identified as the amplitudes of the two quadrature phases of the field.

From the commutation relation (6) we deduce the following relation for the uncertainties $\Delta X_i = \{V(X_i)\}^{1/2}$ in X_1 and X_2

$$\Delta X_1 \Delta X_2 \geq \tfrac{1}{4} \tag{8}$$

A family of minimum uncertainty states is defined by taking the equal sign. One such class of minimum uncertainty states is the coherent states which have $V(X_1) = V(X_2) = \frac{1}{4}$. A broader class of minimum uncertainty states may have unequal variances in each quadrature. These are the so called squeezed states. The condition for squeezing is

$$V(X_i) < \tfrac{1}{4} \qquad i = 1 \text{ or } 2 \tag{9}$$

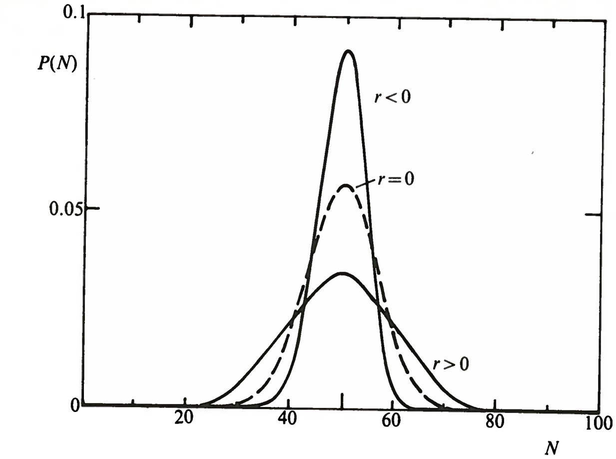

Fig. 3 Photon number distribution for a squeezed state $|\alpha, r\rangle$ ($\alpha = 7$, $r = \pm 0.5$) compared with a coherent state ($r = 0$).

It is sometimes convenient especially for multimode fields to write the condition in terms of the normally ordered variance

$$:V(X_i): < 0 \qquad i = 1 \text{ or } 2 \tag{10}$$

Figure 1 shows a phase space plot of the uncertainties in X_1 and X_2 for a coherent state, a squeezed state and a number state is shown. These error ellipses may be rigorously derived as the contours of the Q function[4].

The time dependence of $E(t)$ including the uncertainty $\Delta E(t)$ is shown in Fig. 2 for *a* a coherent state, *b* a squeezed state with reduced amplitude fluctuations, *c* a squeezed state with reduced phase fluctuations. The corresponding error box for these states at $t = 0$ is also shown.

For a single mode field the variance in one quadrature may be calculated using the Glauber–Sudarshan P representation

$$V(X_1) = \tfrac{1}{4}\left\{1 + \int P(\alpha)[(\alpha + \alpha^*) - (\langle\alpha\rangle + \langle\alpha^*\rangle)]^2 \, d^2\alpha\right\} \tag{11}$$

The condition for squeezing $V(X_1) < \frac{1}{4}$ requires that $P(\alpha)$ be a nonpositive definite function. In this sense squeezing like photon antibunching is a nonclassical property of the electromagnetic field. Note that to derive equation (11) the commutation relation $[a, a^\dagger] = 1$ has been used. If a classical field is assumed from the outset arbitrary squeezing may be obtained in either quadrature. Thus squeezing has a non trivial significance only in the case of quantized fields. A distinction between classical and quantum fields may be obtained from the normally ordered correlation function $g^{(2)}(0)$ which is always ≥ 1 for classical fields (see ref. 20).

Properties of squeezed states

We shall now briefly describe the mathematical properties of squeezed states. A coherent state $|\alpha\rangle$ may be generated by the action of the displacement operator $D(\alpha)$ on the vacuum

$$|\alpha\rangle = D(\alpha)|0\rangle \tag{12}$$

where

$$D(\alpha) = e^{-\frac{1}{2}|\alpha|^2} \, e^{\alpha a^\dagger} \, e^{-\alpha^* a}$$

A squeezed state $|\alpha, \zeta\rangle$ may be generated by first acting with the squeeze operator $S(\zeta)$ on the vacuum followed by the displacement operator $D(\alpha)$ (ref. 21)

$$|\alpha, \zeta\rangle = D(\alpha)S(\zeta)|0\rangle \tag{13}$$

where

$$S(\zeta) = \exp\left(\tfrac{1}{2}\zeta^* a^2 - \tfrac{1}{2}\zeta a^{\dagger 2}\right)$$

and

$$\zeta = r\, e^{i\theta}$$

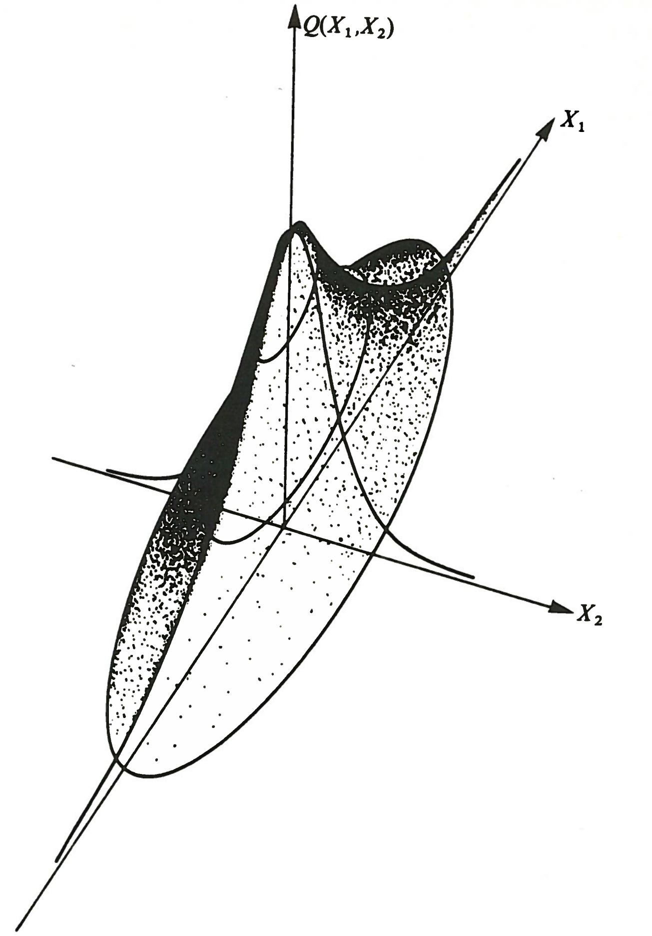

Fig. 4 Q function for a squeezed state. Reproduced with permission from Yuen[4].

An alternative but equivalent characterization of squeezed states has been given by Yuen[4]. We note that whereas a coherent state is generated by linear terms in a and $a^\dagger$ in the exponent a squeezed state requires quadratic terms.

The variances in squeezed state $|\alpha, \zeta\rangle$ are given by

$$\begin{aligned} V(Y_1) &= \tfrac{1}{4}\, e^{-2r} \\ V(Y_2) &= \tfrac{1}{4}\, e^{2r} \end{aligned} \tag{14}$$

where $Y_1 + iY_2 = (X_1 + iX_2)\, e^{-i\theta/2}$ is a rotated complex amplitude so that $2\Delta Y_1$ and $2\Delta Y_2$ represent the length of the minor and major axes of the error ellipse. The mean photon number in the squeezed state $|\alpha, \zeta\rangle$ is

$$\langle n\rangle = |\alpha|^2 + \sinh^2 r \tag{15}$$

Clearly, the variances $V(X_i)$ are independent of the field amplitude α. Thus squeezing is a quantum mechanical effect which may occur in fields with high intensity. In this sense one may say it is a macroscopic quantum effect. This is a significant difference from photon antibunching which is only appreciable for fields with low intensity. There is no general relation between photon antibunching and squeezing, however, we shall consider the limit where the coherent amplitude greatly exceeds the squeezing ($|\alpha|^2 \gg \sinh^2 r$). In this limit we find

$$:V(X_1): = \frac{\alpha^2}{4}(g^{(2)}(0) - 1) = \tfrac{1}{4}(e^{-2r} - 1) \tag{16}$$

where we have chosen α real so that the amplitude is carried by X_1.

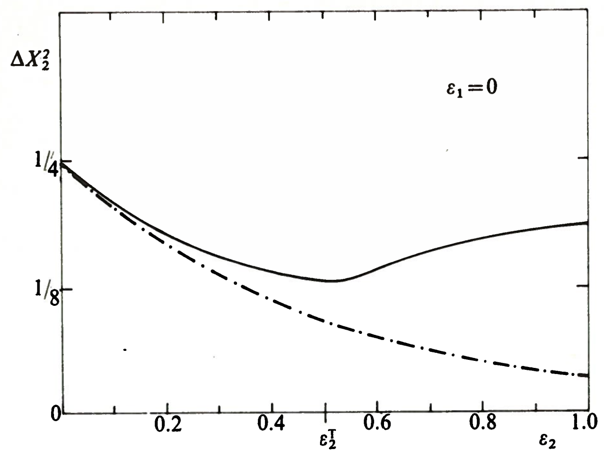

Fig. 5 Squeezing in the parametric oscillator (——) compared with an ideal parametric amplifier (-·-·-·-) as a function of the pump driving field.

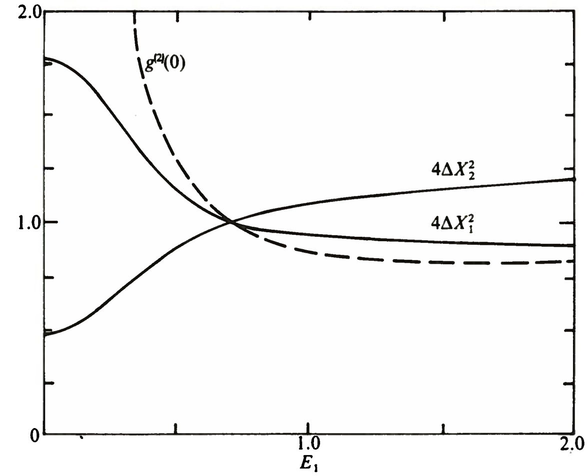

Fig. 6 Photon statistics $g^{(2)}(0)$ and variances ΔX_i^2 for the parametric oscillator as a function of the idler driving field. The pump driving field is held fixed at the threshold value.

For $r>0$ we have a reduction in amplitude fluctuations $(:V(X_1):<0)$ and photon antibunching $(g^{(2)}(0)<1)$ whereas for $r<0$ we have an increase in amplitude fluctuations and photon bunching. Hence in this limit a squeezed state may show either photon bunching or antibunching depending on whether the amplitude fluctuations are increased or reduced. The photon number distribution[4] in a squeezed state $|\alpha, r\rangle$ is plotted in Fig. 3 for $\alpha=7$, $r=\pm 0.5$. We can see that the photon statistics are sub- or super-poissonian depending on whether $r>0$ or $r<0$.

No such simple relation between antibunching and squeezing exists for all values of α. For example in the opposite limit of $\alpha \ll 1$, that is, a squeezed vacuum $|0, r\rangle$,

$$g^{(2)}(0)=1+\frac{\cosh 2r}{\sinh^2 r} \tag{17}$$

Thus the photons in a squeezed vacuum are always bunched irrespective of the sign of the squeeze parameter.

An example of a quantum state which exhibits photon antibunching but no squeezing is a number state $|n\rangle$ for which

$$V(X_1)=V(X_2)=(\tfrac{1}{4})(2n+1) \tag{18}$$

The complete absence of phase information in a number state is clear from the phase space annulus shown in Fig. 1*c*.

As the Glauber–Sudarshan P representation does not exist for a squeezed state we must consider an alternative representation such as the Wigner function, the Q function, or the generalized P function[22,23]. The Q function for a squeezed state is derived in ref. 4. The distribution $Q(X_1, X_2)$ plotted in Fig. 4 as a function of the amplitudes of the two quadratures clearly shows the unequal variances in X_1 and X_2.

Production of squeezed states

There has been no experimental manifestation of squeezed states of light. The requirement to produce a squeezed state may be simply expressed as follows. For a single mode field mix a part of the field with its phase conjugate to produce a new mode b such that

$$b=\mu a+\nu a^{\dagger} \tag{19}$$

where $\mu^2-\nu^2=1$. For mode a in a coherent state the mode b will be in a squeezed state[4]. Thus a scheme involving a phase conjugate mirror appears as one of the favourite candidates for a state squeezer[24]. The above prescription seems very simple, however, the phase conjugate mirror involves a nonlinear interaction, in this case a four-wave mixing interaction. Squeezed states may also be generated by a three-wave mixing interaction as for example in the parametric amplifier[25–27,51,52]. The prototype for these interactions is described by the Hamiltonian,

$$H=\hbar[\chi^{(n)*}(\varepsilon)a^2+\chi^{(n)}(\varepsilon)a^{\dagger 2}] \tag{20}$$

where

$\chi^{(2)}(\varepsilon)=\varepsilon\chi^{(2)}$ (degenerate parametric amplifier)

$\chi^{(3)}(\varepsilon)=\varepsilon^2\chi^{(3)}$ (four-wave mixing)

$\chi^{(n)}$ is the nonlinear susceptibility of the optical medium and ε is the amplitude of the pump field which has been treated classically. This approximate form of the Hamiltonian generates squeezed states with a squeeze parameter given by $r=|2\chi^{(n)}(\varepsilon)t|$. Several objections to this ideal system can be raised. Fluctuations resulting from the quantization of the pump field and the nonlinear medium have been neglected as have vacuum fluctuations associated with any loss process. The vacuum fluctuations will tend to equalize the variances in the two quadratures and hence destroy the squeezing[28–29]. Thus the characteristic damping time of any loss mechanism should be long compared with the interaction time. Phase and amplitude fluctuations in the laser used for the pump may also degrade the squeezing[30]. Phase fluctuations may be compensated for by using part of the pump as the local oscillator in a homodyne detection scheme.

The magnitude of the squeezing is limited by the small values of the nonlinear susceptibility and the interaction time. To increase the interaction time the nonlinear crystal may be placed inside an optical cavity. There is a parametric oscillator configuration where the cavity modes are driven externally by classical fields. An analysis of the cavity must include the cavity losses which tend to destroy the squeezing. Thus there will be a competition between the squeezing produced by the nonlinear interaction and the degradation of the squeezing by the damping. This results in a limiting value to the squeezing attainable in the steady state. An analysis of the degenerate parametric oscillator including the quantization of the pump field has been carried out[31–33]. When only the pump mode is driven by an external field there exists a threshold driving field below which the semiclassical value of the mean field is zero. The squeezing in the idler mode as a function of the pump field amplitude is shown in Fig. 5. As the pump amplitude is increased from zero squeezing appears in the idler mode. However, the squeezing approaches a maximum value corresponding to $V(X_2)=1/8$ close to the threshold value of the pump field, then decreases as the pump power is increased above threshold.

The case where the driving field for the pump mode is held fixed at the threshold value and the driving field for the idler is increased from zero is shown in Fig. 6. For low values of the idler driving field the squeeze parameter is initially positive and amplitude fluctuations are increased, hence we have photon bunching ($g^{(2)}(0)>1$). As the idler driving field is increased the squeeze parameter goes to zero and then becomes negative. Thus we have reduced amplitude fluctuations and hence photon antibunching ($g^{(2)}(0)<1$). This is consistent with the general properties of squeezed states with $|\alpha|^2 \gg \sinh^2 r$ discussed above. This system provides a feasible scheme for detecting squeezed states by making photon correlation measurements directly on the squeezed field. Facility to change the sign of the squeeze parameter and observe the accompanying change of the photon statistics from bunching to antibunching would indicate the presence of a squeezed state.

Other nonlinear intracavity devices have been shown[34,35] to give a maximum squeezing factor not greatly exceeding 2. The coupling of the cavity modes to the vacuum fluctuations of the extracavity modes apparently acts as a counter to the squeezing produced by the nonlinear interaction in a steady-state configuration.

One possibility of avoiding the limitation to squeezing imposed by the vacuum fluctuations entering the cavity is to make one mirror perfectly reflecting. It has been claimed that since the vacuum fluctuations may no longer enter from the second port arbitrary squeezing is in principle attainable (B. Yurke, personal communication).

Another way to avoid the problem of vacuum fluctuations is to revert to the parametric amplifier configuration where the cavity losses no longer have a role. The parametric amplifier is a travelling wave phase matched interaction and the Hamilton equation (20) which only includes a single mode is not appropriate. A multimode analysis[36] of a travelling wave parametric amplifier indicates that a reduction in squeezing over the single mode case may occur for the non degenerate amplifier. This reduction in squeezing is caused by the contribution from non-resonant modes whose axes of squeezing become misaligned with respect to the resonant mode.

Another possible system for producing squeezed states is a two-photon laser due to the quadratic nature of the field interaction. A laser, however, is an active system in which the atoms are pumped to the excited state and may consequently decay by spontaneous emission. Calculations using a two-level model for the atomic medium reveal that any potential squeezing is destroyed by the fluctuations resulting from spontaneous emission[37,38].

It is clear, therefore, that a phase sensitive nonlinear interaction in a passive medium is required to produce squeezed states. Predictions of squeezing in a variety of nonlinear optical processes have now been made, for example the free electron laser[39], second harmonic generation[40,41], the single atom-single field mode interaction[42], and multiphoton absorption[43]. The prediction of squeezing in four wave mixing[24] has attracted the interest of experimentalists[42]. An analysis of the effect of atomic fluctuations in four-wave mixing based on a two-level atomic medium reveals that for the atoms driven near saturation or close to resonance the spontaneous emission will destroy the squeezing[45]. For significant squeezing the driving fields should be of low intensity and sufficiently far from resonance so as not to saturate the atoms.

Another system with somewhat different characteristics is squeezing in resonance fluorescence from a two-level atom[46]. Resonance fluorescence differs from many of the systems discussed above as it involves many modes of the radiation field. Resonance fluorescence deserves attention as it is the only system in which photon antibunching has been observed[12,13].

We consider a two-level atom driven by a coherent driving field. The product of the amplitude of the driving field and the dipole moment of the atom is characterized by the Rabi frequency. We denote the Rabi frequency normalized by the natural linewidth of the atom by Ω. The driving field may have a detuning with respect to the atomic transition. We shall use δ to characterize the detuning normalized by the natural linewidth.

The condition for squeezing in a field may best be expressed in terms of the normally ordered variances which do not include the contribution from the vacuum fluctuations. For squeezing in either quadrature ($E_1=(E^{(+)}+E^{(-)})/2$, $E_2=(E^{(+)}-E^{(-)})/2i$) of the field we require

$$:V(E_i):<0 \qquad i=1 \text{ or } 2 \tag{21}$$

We calculate the squeezing in the components of the fluorescent field in the direction along and perpendicular to the mean field. The variance in the component E_1' in the direction of the mean field is

$$:V(E_1'):=\frac{-\lambda}{4}\frac{(1+\delta^2-\Omega^2)}{1+\delta^2+\Omega^2}\Omega^2 \tag{22}$$

where λ is a constant.

Thus we find squeezing in this component provided $\Omega^2<1+\delta^2$. No squeezing occurs in the component orthogonal to the mean field. The reduced amplitude fluctuations occurring for $\Omega^2<1+\delta^2$ is consistent with the observed fact that the fluorescent light is antibunched. We note that the fluorescent light is also antibunched in the strong field limit $\Omega^2>1+\delta^2$ where there is no squeezing. In this limit the characteristics of the fluorescent light resemble a number state.

Detection of squeezed states

Proposals to measure the variances in the quadrature phases of a light field suggest homodyning or heterodyning the signal with a local oscillator which gives the necessary phase dependence followed by a photon counting measurement. Such measurements are feasible with existing technology. (For further details of such a measurement scheme see refs 6, 7, 20.)

The signal field is homodyned with a local oscillator which is assumed to be in a coherent state. The complex amplitude of the local oscillator may be written as $\varepsilon=|\varepsilon|\,e^{i\theta}$ where θ is the phase of the local oscillator with respect to the signal field. In the limit where the amplitude of the local oscillator greatly exceeds the amplitude of the signal field the photon statistics of the combined field are directly related to the normally ordered variance of the signal field. Assuming a perfect detector efficiency it may be shown that[6,7,20]

$$\begin{aligned} V(n)-\langle n\rangle &= 4|\varepsilon|^2{:}V(E_1){:} \quad \text{if } \theta=0 \\ &= 4|\varepsilon|^2{:}V(E_2){:} \quad \text{if } \theta=\pi/2 \end{aligned} \tag{23}$$

Thus by changing the phase of the local oscillator a measurement of the photon statistics yields the normally ordered variance in E_1 ($\theta=0$) and the normally ordered variance in E_2 ($\theta=\pi/2$). A change of photon statistics from sub- to super-poissonian as θ is varied will indicate the presence of squeezing.

Such measurements impose a stringent requirement on the relative phase stability between the local oscillator and the signal. Yuen and Chan[47] have recently suggested that photon number fluctuations in the local oscillator may be eliminated using a balanced detector scheme developed in the microwave region[48].

Another way to detect a squeezed state is by a direct photon correlation measurement if one has the facility to vary the sign of the squeeze parameter. The presence of a squeezed state is indicated by a change of photon statistics from bunching to antibunching as the squeeze parameter is varied. An example of such a system is the parametric oscillator with two driving fields discussed earlier. This method obviates the need for homodyning the signal with a local oscillator.

Applications of squeezed states

Squeezed states have several potential applications, one, for example, is in optical communication systems. In a proposed scenario information would be transmitted in the quadrature of the field with reduced quantum fluctuations. An enhanced signal-to-noise ratio could then be obtained in the quantum noise limited regime over information sent using coherent light beams. The application of squeezed states in optical communications systems is discussed in refs 6 and 7.

Similar considerations hold in the amplification of signals. Noise is necessarily added in the amplification process, however, if a suitable phase sensitive amplifier is used the noise may be added preferentially to the quadrature not carrying information. This leaves the amplification of the quadrature carrying the information essentially noise free.

Interferometric techniques to detect very weak forces such as gravitational radiation experience limitations on sensitivity due to quantum noise arising from photon counting and radiation pressure fluctuations. These sources of noise may be interpreted as arising from the beating of the input laser with the vacuum fluctuations entering the unused port of the interferometer. It turns out that these two different noise sources arise from fluctuations in the two different quadrature phases of the vacuum entering the unused input port. It has been suggested by Caves[21] that injecting a squeezed state into the unused input port will reduce one or other of the two sources of noise depending on which quadrature is squeezed.

Another intriguing application of squeezed states is in an optical waveguide tap. Shapiro has shown that a high signal-to-noise ratio may be obtained using a squeezed state in an optical waveguide to tap a signal carrying waveguide[49]. This may be achieved with very low energy loss from the signal thus offering the possibility of permitting optical data bus technology to reach multikilometre path lengths with many user sites but no repeaters.

Conclusions

The field of quantum optics has been an active field of research since the early 1960s. However, much which has been discussed under this heading could more correctly be described as nonlinear optics as no quantization of the electromagnetic field is necessary. Very few features which are explicitly a result of quantization of the field have been observed—photon antibunching being one exception. Squeezed states represent a class of quantum states for which no classical analogue exists, hence their detection would be of fundamental interest.

The achievements of quantum optics have been based on the measurement of photon correlation functions of the electromagnetic field. We now seem to be on the verge of an era where a new class of measurements on the phase dependent correlation functions of the electromagnetic field will be possible. This will enable information on the electromagnetic field to be obtained which was not accessible from photon correlation measurements. Such measurements based on homodyning or heterodyning the field with a local oscillator appear feasible with current technology. The presence of a squeezed state will be indicated by the observation of sub-poissonian photon statistics in such a phase sensitive detection process.

Present efforts are directed towards methods of generating a squeezed state. While a proof in principle of the existence of squeezed states seems possible in, for example, resonance fluorescence from a two-level atom or an intracavity nonlinear optical interaction the magnitude of the squeezing obtained in such systems is small. To obtain appreciable squeezing one must look to either a single pass device with a high nonlinearity and low losses or possibly to a cavity with a single input/output port which prevents the vacuum fluctuations entering as in the two port cavity.

Should a device be found to give a light field with significant squeezing the potential applications are attractive. These applications lie on the frontier of technology in quantum noise limited situations. For example, a squeezed light field could be used in an optical communication system where the information is carried by the quadrature with reduced quantum fluctuations. This would enable a better signal-to-noise ratio to be attained than using conventional laser sources which are limited by the quantum noise of a coherent state. The general concept of squeezed states with their phase dependence of quantum noise has important implications in quantum amplifier theory and ultrasensitive electronics such as required for the detection of gravitational radiation. While no experimental observation of squeezed states has yet been reported this is a goal well worth achieving both from a fundamental point of view and in consideration of the applications that will follow.

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