Exercises

Exercise 1

a- Show that the squeezing operator,

$$S(r,\phi) = \exp\left[\frac{r}{2}\left(e^{-2i\phi}a^2 - e^{2i\phi}a^{\dagger 2}\right)\right]$$

may be put in the normally ordered form,

$$S(r,\phi) = (\cosh r)^{-1/2}\exp\left(-\frac{\Gamma}{2}a^{\dagger 2}\right)\exp[-\ln(\cosh r)\, a^\dagger a]\exp\left(\frac{\Gamma^*}{2}a^2\right)$$

where $\Gamma = e^{2i\phi}\tanh r$.

b- Express $S(r,\phi)|0\rangle$ in terms of Fock states.

Exercise 2

If $|X\rangle$ is an eigenstate of the operator $X \doteq \frac{1}{\sqrt{2}}(a^\dagger + a)$ find $\langle X|\psi\rangle$ in the cases (a) $|\psi\rangle = |\alpha\rangle$; (b) $|\psi\rangle = |\alpha, r\rangle$.

Exercise 3

a- Having in mind the properties of $S(r,\phi)$ and $D(\alpha)$, evaluate the ground state and the ground-state energy of the following Hamiltonians:

$$H_1 = \omega_0 a^\dagger a + \varepsilon^* a + \varepsilon a^\dagger$$

$$H_2 = \omega_0 a^\dagger a + \eta^* a^2 + \eta a^{\dagger 2}$$

(assume that $|\eta|/\omega_0 \leq 1/2$)

b- What is the expectation value $\langle a^\dagger a\rangle$ for the ground state of H_1 and H_2?

Exercise 4

Calculate the mean intensity at the screen when the two slits of a Young's interference experiment are illuminated by the two photon state $(b^\dagger)^2|0\rangle/\sqrt{2}$ where $b = (a_1 + a_2)/\sqrt{2}$ and a_i is the annihilation operator for the mode radiated by slit i.