1. Derive the expression for the penetration depth of a neutral into a plasma, given in the lectures as

$$\lambda_n = \frac{v_n}{n_e \langle \sigma v_e \rangle}. \tag{1}$$

   Hint: Start by considering the neutrals to be a stationary target with respect to the electrons, and use the previously derived result for an electron's mean free path $\lambda = 1/n_t\sigma$ before an ionizing collision. From there you can deduce the mean time between ionizing collisions.

2. Consider a plasma having an electron density $n_e = 1 \times 10^{11}\,\mathrm{cm}^{-3}$ in a magnetic field whose gradient is $\nabla B = 1\,\mathrm{T/m}$. A 6.4 GHz microwave is inbound to the plasma. Calculate the distance in millimeters between the UHR and ECR resonances. (Often the wave travels at an acute angle with respect to the magnetic field, enabling both the upper hybrid -and electron cyclotron resonances.)

3. For the same plasma as before, suppose that the microwave is inbound perpendicular to the magnetic field. Plot the square of the refractive index, n^2, as a function of distance (in mm) from the resonance point.

4. Same as before, but for a magnetic field with a gradient of $2\,\mathrm{T/m}$. For which value of ∇B do you think the electron heating is more effective? Why?

5. Consider 14 and 18 GHz ECR ion sources, where $\nabla B = 5\,\mathrm{T/m}$ and the microwave is impinging into the plasma parallel to the field lines. For plasma densities, $n_e = 10\,\%$ and $n_e = 50\,\%$ of the critical density, what fraction of the microwave power has been absorbed by the plasma in each case? The critical density is defined as the density for which $\omega_{\mathrm{RF}} = \omega_{pe}\left(n_{e,\mathrm{critical}}\right)$.

- BONUS (1 p.): In an Electron Cyclotron Resonance Ion Source (ECRIS), the plasma discharge is sustained via continuous input of high frequency microwaves. The microwaves heat up electrons (confined within a magnetic bottle), whose increased temperature allows the plasma to be formed due to ionizing collisions between the electrons and atoms of the low pressure gas, which is being fed into the plasma chamber. The frequency of the microwaves required by the ECR heating method lies in the GHz range. **Find out:** *How are GHz range microwaves formed in the laboratory, and how can they be guided into the plasma?*