Home work assignment 7 : magnetism

(Dated: January 10, 2020)

Problem 1: Ferromagnetism of interacting electronsin Hartree-Fock approximation
Consider a polarized electron gas in which $N_\pm$ denotes the number of electrons with spin-up/down. Let us define $r_s = r_0/a_0$, where $r_0 = [3/(4\pi n)]^{1/3}$ and $a_0 = \hbar^2/(me^2)$, n is electron density.

- (a) Find the ground state energy to the first order in the interaction potential as a function of $N = N_+ + N_-$ and the polarization $\zeta = (N_+ - N_-)/N$.
- (b) Show that the ferromagnetic state $\zeta = 1$ represents a lower energy than the non-magnetic state $\zeta = 0$ if $r_s > (2\pi/5)(9\pi/4)^{1/3}(2^{1/3} + 1) = 5.45$.

Problem 2: Heisenberg model of ferro- and antiferro-magnetism The Heisenberg "exchange" Hamiltonian for two spins is given by

$$\hat{H}_{ex} = -2J\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \tag{1}$$

where $\boldsymbol{S} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ is a vector of Pauli matrices.

- (a) Find the spectrum of $\hat{H}$ and the spin structure of wave function corresponding to the minimal energy state.
- (b) Show that the spectrum of interacting two-particle system

$$\hat{H} = H_0(\boldsymbol{r}_1) + H_0(\boldsymbol{r}_2) + \frac{e^2}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|} \tag{2}$$

$$H_0(\boldsymbol{r}) = -\frac{\hbar^2\nabla^2}{2m} - |U_0|[\delta(\boldsymbol{r} - \boldsymbol{a}) + \delta(\boldsymbol{r} + \boldsymbol{a})] \tag{3}$$

with $\boldsymbol{a} = a\boldsymbol{x}$. has the spectrum coinciding with Heisenberg model. Use the fact that the electronic many-body wave function should be antisymmetric by interchanging coordinate and spin simultaneously.