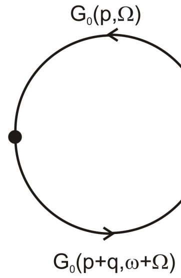
Homework assignment 3: Random phase approximation, Feynman rules at zero-temperature(Chapter 7 from Coleman' textbook)

(Dated: January 9, 2020)

- **Problem 1:** Write the analytical expression for the polarization operator $\Pi(\omega, \boldsymbol{q})$ at zero temperature according



  to the diagram shown in the figure.

- **Problem 2:** Random phase approximation (RPA).

  Using the polarization operator introduced in Problem 1 construct the perturbation expansion for the renormalized interaction potential and derive the expression for the generalized dielectric function $\kappa(q, \omega)$, renormalizing the interaction : $U(q, \omega) = U_0(q, \omega)/\kappa(q, \omega) = U_0(q, \omega)/[1 - \Pi(q, \omega)U_0(q, \omega)]$.

- **Problem 3:** Coulomb screening.

  (i) In the static limit, derive the Lindhard dielectric function $\varepsilon(q) = \kappa(q, 0)$ for the Coulomb interaction. Show that the singularity of $\varepsilon(q)$ at $q = 2k_F$ leads to the Freidel oscillations of the electric potential around the point charge in metals.

  (ii) Find the screening of smooth potentials with the Fourier components $q \ll k_F$. Derive the Tomas-Fermi screening length.

- **Problem 4:** Effective mass remormalization. Using the screened Coulomb interaction in the static limit $U(q, \omega) = U_0(q, \omega)/\kappa(q, 0)$, calculate the Hartree-Fock self-energy. Find the quasiparticle velocity and effective mass re-normalization due to the screening.