Section 4-5 Perturbation theory: Feynman Diagrams

So far we have not seen any useful application of GF. Of course if we know GF we can calculate any observable. But how to calculate it especially for systems which include interaction between particles? We can use perturbation theory. The reason why GF were introduced is that there exist a technology of computing perturbation series of infinite order for GFs. It is possible to present perturbation expansion as a set of graphs – Feynman diagrams.

As in some medieval paintings there is a strict set of rules for both drawing and reading those images.

Perturbation theory. We want to calculate GF $G = -i\langle 0|T\psi'\psi^+|0\rangle$

As we have discussed before, in order to built perturbation theory we can use interaction representation of field operators

1) Hamiltonian: $H = H_0' + W$ where $H_0' = H_0 - \mu N$

2) I.R. $\psi_H(r,t) = S^+(t,t_0)\,\psi_S(r)\,S(t,t_0)$ $\quad \psi_I(r,t) = e^{iH_0't}\psi_S e^{-iH_0't}$

$S(t,t_0) = T\exp\left(-i\int_{t_0}^{t} H(t)\,dt\right)$

3) Perturbation in I.R. $W_I = \exp(iH_0't)\,W\exp(-iH_0't)$ – it becomes time-dependent

4) Hamiltonian in I.R. $H_I = W_I \equiv \exp(iH_0't)\,W\exp(-iH_0't)$

5) Evolution operator $\hat{S}(t,t_0) = T\exp\left(-i\int_{t_0}^{t}\hat{W}_I(t)\,dt\right)$ – prove HW!

which means the symbolic form of the sum

$$\hat{S}(t,t_0) = \prod_{t_i=t_0}^{t}\left(1 - i\delta t\, W_I(t_i)\right) = \prod_{t_i=t_0}^{t}\exp\left(-i\delta t\, W_I(t_i)\right)$$

The evolution operator is

$$\hat{S}^{-1} = \hat{S}^+ \quad \text{– unitarity}$$

$$\hat{S}(t_3,t_2)\,\hat{S}(t_2,t_1) = \hat{S}(t_3,t_1)$$

$$\hat{S}^{-1}(t_2,t_1)\,\hat{S}^{-1}(t_3,t_2) = \hat{S}^{-1}(t_3,t_1)$$

Adiabatic principle

In the definition of GF $G = -i\langle 0|T\psi'\psi^+|0\rangle$ the average is taken over $|0\rangle$ = Heisenberg's (time-independent!) wave function of the interacting system.

In general it is very difficult to find the wave function of many-particle system. Any approximation we will make will be almost orthogonal to the exact wave function. This is because if we make even a tiny mistake approximating the single particle wave function α so that its projection to the exact single-particle state is $(1-\alpha)$. Then for many-particle state the projection is $(1-\alpha)^N \xrightarrow[N\to\infty]{} 0$ (2)

Hence we need to know exactly the wave function $|0\rangle$
The only system for which we can find exact wave function is that of non-interacting particles.

Therefore we make an assumption that the interaction between particles switches on adiabatically

$$W(t=-\infty)=0$$

Then the state $|0\rangle$ is the ground state of non-interacting particles. Since it is the Heisenberg wave function it is time-independent and remains the same.

However in the interaction picture it will evolve

$$\Phi_I(t) = \hat{S}(t,-\infty)\,\Phi_H$$

We can rewrite the Heisenberg's operator as follows

$$\hat{\psi}_H = \hat{S}^{-1}(t,-\infty)\,\hat{\psi}_I\,\hat{S}(t,-\infty)$$

We choose here the moment $t=-\infty$ when the interaction and Heisenberg's representations coincide.

Now we can apply interaction representation to the G.F.

$$iG(x_1,x_2) \overset{t_1>t_2}{=} \langle \hat{\psi}(t_1)\,\hat{\psi}^+(t_2)\rangle = -\langle \hat{S}^{-1}(t_1,-\infty)\,\hat{\psi}_I(t_1)\,\hat{S}(t_1,-\infty)\times$$
$$\times S^{-1}(t_2,-\infty)\,\hat{\psi}_I^+(t_2)\,\hat{S}(t_2,-\infty)\rangle = \langle \hat{S}^{-1}(\infty,-\infty)\,\underbrace{\hat{S}(\infty,t_1)\,\hat{\psi}_I(t_1)\cdot}_{\text{in chronological order}}$$
$$\cdot\,\hat{S}(t_1,t_2)\,\hat{\psi}_I^+(t_2)\,\hat{S}(t_2,-\infty)\rangle = -\langle S^{-1}\,T(\psi_I(t_1)\,\psi_I^+(t_2)\,S)\rangle$$

where we denote $\hat{S} = S(+\infty,-\infty) = T\exp\left(-i\int_{-\infty}^{\infty}\hat{W}_I(t)\,dt\right)$

calculations for $t_2>t_1$ are analogous and the final result is correct for both cases

$$iG(x_1,x_2) = \langle S^{-1}\,T(\psi_I(t_1)\,\psi_I^+(t_2)\,\hat{S})\rangle$$

This transformation does not depend on the state $|0\rangle$. However if this state is <u>ground state</u> we can simplify the result further. For this let us notice that when we switch on and off the interaction slowly the system remains in the ground state. Hence the action of operator $\hat{S}$ reduces to a phase shift

$$\hat{S}|0\rangle = e^{i\alpha}|0\rangle \Rightarrow e^{i\alpha} = \langle 0|\hat{S}|0\rangle \qquad (3)$$

Hence we get the GF

$$iG(x_1,x_2) = \frac{1}{\langle \hat{S}\rangle} \langle T(\hat{\psi}_I(x_1)\ \hat{\psi}_I^+(x_2)\ \hat{S})\rangle \qquad (*)$$

The average here is taken over the ground state of non-interacting system!

Since for non-interacting particles interaction and Heisenbergs representations coincide then the expression

$$iG^{(0)}(x_1,x_2) = \langle T\psi_I(x_1)\ \psi_I^+(x_2)\rangle$$

is the GF of non-interacting particles.

Diagram technique for Fermi-systems

The meaning of expression (*) is that it allows to write down expansion by the powers of perturbation term W

$$\langle T\ \hat{\psi}_I(x)\ \hat{\psi}_I^+(x')\ \hat{S}\rangle = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} dt_1 \dots \int_{-\infty}^{\infty} dt_n \langle T\ \hat{\psi}_I(x)\ \hat{\psi}_I(x')\ \hat{W}_I(t_1)\dots \hat{W}_I(t_n)\rangle$$

$$\langle \hat{S}\rangle = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} dt_1 \dots \int_{-\infty}^{\infty} dt_n \langle T\ \hat{W}_I(t_1) \dots \hat{W}_I(t_n)\rangle$$

Since all operators can be expressed throug field operators $\hat{\psi}_I$ Hence to calculate expansion terms we need to calculate averages over the ground state from T-ordered products of several $\hat{\psi}_I$ operators.

Such calculations are facilitated with the help of <u>diagramm technique</u> which is based on the Wick's theorem that we have already discussed when we calculated ground state energy of electron gas.

I would like to remind once again that we in fact will discuss not the Wick's theorem itself but a weaker statement for the averages in thermodynamic limits.

Wick's theorem

We have already calculated products of 4 operators

$$\langle \psi_1^+ \psi_2^+ \psi_3 \psi_4 \rangle = -\langle \psi_1^+ \psi_3 \rangle \langle \psi_2^+ \psi_4 \rangle + \langle \psi_1^+ \psi_4 \rangle \langle \psi_2^+ \psi_3 \rangle$$

This result can be generalized as follows

1) First of all the permutation of operators on both sides doesn't change result. Hence we can place them in chronological order

$$\langle T \psi_1^+ \psi_2^+ \psi_3 \psi_4 \rangle = \langle T \psi_1^+ \psi_4 \rangle \langle T \psi_2^+ \psi_3 \rangle - \langle T \psi_1^+ \psi_3 \rangle \langle T \psi_2^+ \psi_4 \rangle$$

2) Second we can consider arbitrary (even) number of operators

$\langle T \psi_1^+ \ldots \psi_n^+ \psi_1 \ldots \psi_n \rangle$ = sum of all T products of all possible $\psi \psi^+$ pair averages. The sign of each term is determined by the parity of the number of permutations required to bring all paired operators together. Note that the pair average is just the unperturbed Green's function

$$i G^0_{\alpha,\beta}(x_1 t_1, x_2 t_2) = \langle T \psi_\alpha(x_1 t_1) \psi_\beta^+(x_2 t_2) \rangle$$

Now we have a clear algorythm.

1. Expand the time-ordered exponent by powers of W
2. Average each term using Wick's theorem
3. Map the resulting algebraic expressions to the graphic represent.

Usually the diagrams come first.

$$\hat{S} = T\exp\left(-i\int_{-\infty}^{\infty} \hat{W}(t)\,dt\right) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} dt_1 \cdots \int_{-\infty}^{\infty} dt_n\, \hat{W}(t_1) \cdots \hat{W}(t_n)$$

Let us consider a particular example of a two-body interaction

$$W(t) = \frac{1}{2} \sum_{\alpha_1,\alpha_2} \int d^3r_1\, d^3r_2\, \psi^+_{\alpha_1}(r_1,t)\, \psi^+_{\alpha_2}(r_2,t)\, U(\bar r_1 - \bar r_2)\, \psi_{\alpha_2}(r_2,t)\, \psi_{\alpha_1}(r_1,t)$$

For convenience it is useful to introduce

$$U(1-2) \equiv U(r_1 - r_2)\, \delta(t_1 - t_2)$$

and denote the whole set $X = (\alpha, t, \bar r)$

Now let us calculate GF to the first order in W

$$\langle S \rangle = 1 - i \int_{-\infty}^{\infty} dt\, \langle T\, W_I(t) \rangle = 1 - \frac{i}{2} \int dx_1\, dx_2\, U(x_1 - x_2) \langle T \psi^+(x_1) \psi^+(x_2) \psi(x_2) \psi(x_1) \rangle$$

$$\langle T\, \psi(x_1) \psi^+(x_2) S \rangle = \langle T\, \psi(x_1) \psi^+(x_2) \rangle -$$

$$- \frac{i}{2} \int dx_3\, dx_4\, U(x_3 - x_4) \langle T \psi^+(x_3) \psi^+(x_4) \psi(x_4) \psi(x_3) \psi(x_1) \psi^+(x_2) \rangle$$

This was step 1. Now we should apply Wick's theorem.

1) $\langle T\, \psi^+(x_3) \psi^+(x_4) \psi(x_4) \psi(x_3) \psi(x_1) \psi^+(x_2) \rangle = - \langle T \psi^+(x_3) \psi(x_4) \rangle \langle T \psi^+(x_4) \psi(x_3) \rangle \langle T \psi(x_1) \psi^+(x_2) \rangle$

2) $\langle T\, \psi^+(x_3) \psi^+(x_4) \psi(x_4) \psi(x_3) \psi(x_1) \psi^+(x_2) \rangle = + \langle T \psi^+(x_3) \psi(x_3) \rangle \langle T \psi^+(x_4) \psi(x_4) \rangle \langle T \psi(x_1) \psi^+(x_2) \rangle$

3) $\langle T\, \psi^+(x_3) \psi^+(x_4) \psi(x_4) \psi(x_3) \psi(x_1) \psi^+(x_2) \rangle = - \langle T \psi^+(x_3) \psi(x_1) \rangle \langle T \psi^+(x_4) \psi(x_4) \rangle \langle T \psi(x_3) \psi^+(x_2) \rangle$

4) $\langle T\, \psi^+(x_3) \psi^+(x_4) \psi(x_4) \psi(x_3) \psi(x_1) \psi^+(x_2) \rangle = + \langle T \psi^+(x_3) \psi(x_4) \rangle \langle T \psi^+(x_4) \psi(x_1) \rangle \langle T \psi(x_3) \psi^+(x_2) \rangle$

5) $\langle T\, \psi^+(x_3) \psi^+(x_4) \psi(x_4) \psi(x_3) \psi(x_1) \psi^+(x_2) \rangle = + \langle T \psi^+(x_3) \psi(x_1) \rangle \langle T \psi^+(x_4) \psi(x_3) \rangle \langle T \psi(x_4) \psi^+(x_2) \rangle$

6) $\langle T\, \psi^+(x_3) \psi^+(x_4) \psi(x_4) \psi(x_3) \psi(x_1) \psi^+(x_2) \rangle = - \langle T \psi^+(x_3) \psi(x_3) \rangle \langle T \psi^+(x_4) \psi(x_1) \rangle \langle T \psi(x_4) \psi^+(x_2) \rangle$

1) $- \langle T\, \psi(x_4) \psi^+(x_3) \rangle \langle T\, \psi(x_3) \psi^+(x_4) \rangle \langle T\, \psi(x_1) \psi^+(x_2) \rangle = + i\, G(x_4 x_3)\, G(x_3 x_4)\, G(x_1 x_2)$ disconnected

2) $\langle T\, \psi(x_3) \psi^+(x_3) \rangle \langle T\, \psi(x_4) \psi^+(x_4) \rangle \langle T\, \psi(x_1) \psi^+(x_2) \rangle = - i\, G(x_3 x_3)\, G(x_4 x_4)\, G(x_1 x_2)$

3) $- \langle T\, \psi(x_1) \psi^+(x_3) \rangle \langle T\, \psi(x_4) \psi^+(x_4) \rangle \langle T\, \psi(x_3) \psi^+(x_2) \rangle = + i\, G(x_1 x_3)\, G(x_4 x_4)\, G(x_3 x_2)$ W

4) $\langle T\, \psi(x_4) \psi^+(x_3) \rangle \langle T\, \psi(x_1) \psi^+(x_4) \rangle \langle T\, \psi(x_3) \psi^+(x_2) \rangle = - i\, G(x_4 x_3)\, G(x_1 x_4)\, G(x_3 x_2)$ V

5) $\langle T\, \psi(x_1) \psi^+(x_3) \rangle \langle T\, \psi(x_3) \psi^+(x_4) \rangle \langle T\, \psi(x_4) \psi^+(x_2) \rangle = - i\, G(x_1 x_3)\, G(x_3 x_4)\, G(x_4 x_2)$ V

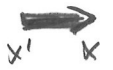
6) $- \langle T\, \psi(x_3) \psi^+(x_3) \rangle \langle T\, \psi(x_1) \psi^+(x_4) \rangle \langle T\, \psi(x_4) \psi^+(x_2) \rangle = i\, G(x_3 x_3)\, G(x_1 x_4)\, G(x_4 x_2)$ W

x_1 x_2 are external coordinates, there is no integration by x_1, x_2

Thus the terms 1) and 2) contain disconnected GF $G(x_1 x_2)$

Now we are ready to the last step – graphical representation of perturbation series.

Feynman rules for scalar electron-electron interaction.



$i G(x,x') \equiv i G_{\alpha\beta}(r,t,r',t')$ Causal GF

$i G^0(x,x')$ Unperturbed causal GF

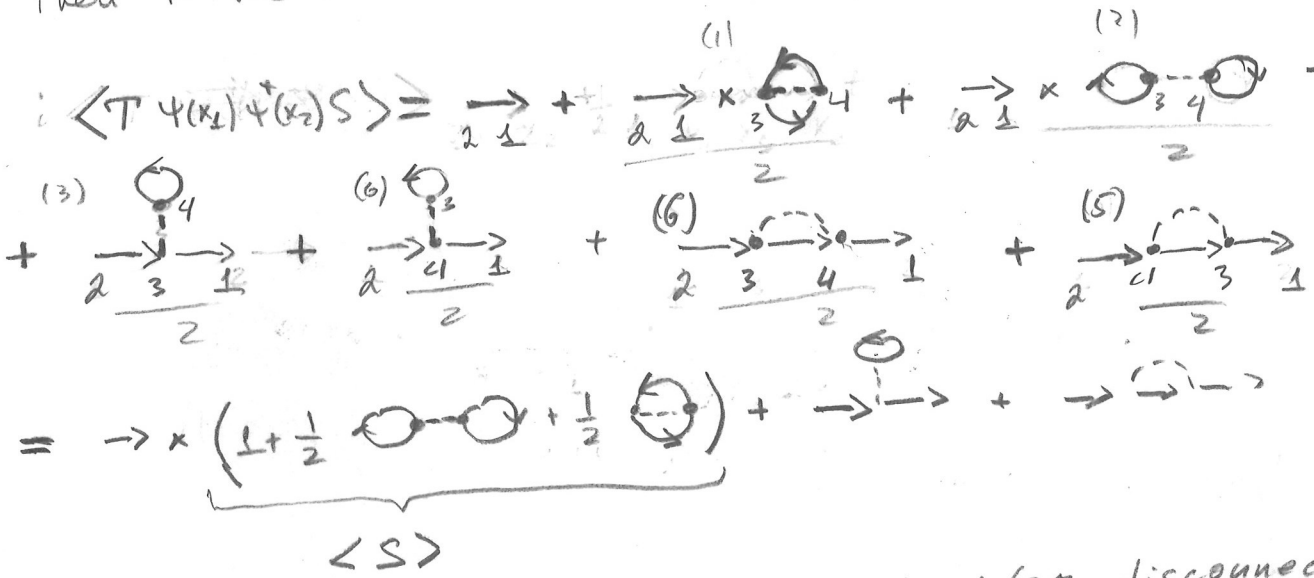
$-i U(1-2) \equiv -i U(x_1-x_2)\,\delta(t_1-t_2)$ Interaction potential

$n^0 \equiv -i G(x_1,x_1)$ Unperturbed electron density

$n_0 = (2m\mu)^{3/2}/3\pi^2$

The integration over all intermediate coordinates and times summation over spin indices is implied

Then to the second order we get

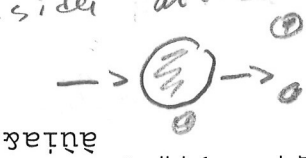


It is easy to check that the second-order disconnected terms are cancelled by the $\langle S\rangle$ term in the denominator.

This applies to all orders of perturbation theory.

<u>Cancellation theorem</u>: All disconnected diagrams appearing in the perturbation series for the GF exactly cancel from its numerator and denominator.

The idea of proof is to consider all diagrams that contain a particular connected part



and all possible disconnected parts

$$iG = \ldots + \frac{(-i)^n}{n!}\langle T\,\psi(t_1)\psi^+(t_2)\,W(t_1)\ldots W(t_m)\,W(t_{m+1})\ldots W(t_n)\rangle + \ldots$$

$$iG = \dots + \frac{(-i)^n}{n!} \int\int dt_1 \dots dt_n \, \langle T\, \psi(x)\psi^+(x')\, W(t_1)\dots W(t_m)\rangle \langle W(t_{m+1})\dots W(t_n)\rangle$$

we can shuffle $W(t_i)$ between two brackets. The number of equivalent terms is $\frac{n!}{m!(n-m)!}$

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Suming over n at fixed m we have

$$iG = \dots + \frac{(-i)^m}{m!} \langle T\, \psi(x)\psi(x')\, W(t_1)\dots W(t_m)\rangle \cdot \underbrace{\sum_{n=m}^{\infty} \frac{(-i)^{n-m}}{(n-m)!} \langle T\, W(1)\dots W(n-m)\rangle}_{\langle S\rangle}$$

equals to the exact denominator

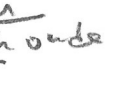
Therefore we can in general forget about denominator and consider only diagrams which are connected to the external lines. $iG(x,x') = \langle T\, \psi(x)\psi^+(x')\rangle_c \leftarrow$ connected

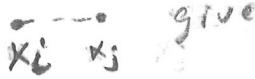
Futhermore any connected term of m-th order there will be exactly $2^m m!$ identical contributions due to the rearrangements of times in the product $\langle T\, \psi(x)\psi(x')\, W(t_1)\dots W(t_n)\rangle$

~~Moreover the factor $\frac{1}{2^n}$ is also cancelled.~~

Reason

n-th order diagram has n dashed lines

1)  gives the same this cancels $1/2$

2)  gives the same as this cancels $1/2!$

Therefore We can calculate parts of diagram independently of the overall structure. Otherwise the order-dependent prefactors would spoil everything.

Example

Then the general rules are as follows.

1. Draw all topologically distinct connected Feynman diagrams
2. Decode them according to the table
3. Multiply every diagram by $(-1)^F$ where F is the number of closed loops with more than one vertex, consisting of fermionic lines. Bubbles no = ○ do not count.

Feynman rules in momentum space

- Consider spatially homogeneous system $G(x_1, x_2) = G(x_1 - x_2)$

$$G(k_1, k_2) = G(k_1 + k_2)\, \delta^4(k_1 - k_2)\, (2\pi)^4$$

- Fourier transform

$$G(x_1 - x_2) = \int \frac{d^4k}{(2\pi)^4}\, G(k)\, e^{ik(x_1 - x_2)} \qquad k = (\mathbf{k}, +\omega)$$

$$kx = \mathbf{k}\mathbf{p} - \omega t$$

- $$V(x_1 - x_2) = \int \frac{d^4q}{(2\pi)^4}\, V(q)\, e^{iq(x_1 - x_2)}$$

Let's calculate 1st order corrections

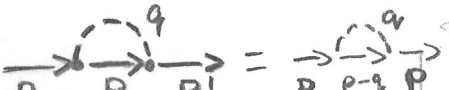
$$= -\int G^0(x - x_1)\, G^0(x_1 - x_2)\, G^0(x_2 - x')\, V(x_1 - x_2)\, d^4x_1\, d^4x_2$$

$$= -(2\pi)^{-16} \int G^0(p_1)\, G^0(p_2)\, G^0(p_3)\, V(q)\, e^{ip_1(x - x_1) + ip_2(x_1 - x_2) + ip_3(x_2 - x') + iq(x_1 - x_2)}\, dp_{1,2,3}\, dq\, dx_1\, dx_2$$

$$= -(2\pi)^{-8} \int G^0(p_1)\, G^0(p_2)\, G^0(p_3)\, V(q)\, \delta(p_1 - p_2 - q)\, \delta(p_2 + q - p_3)\, e^{ip_1 x - ip_3 x'}\, dp_{1,2,3}\, dq\, dx_1\, dx_2$$

Consider Fourier components $p_1 = p$ $p_3 = p'$

$$iG^{(1)} = -\int G^0(p)\, G^0(p_2)\, G^0(p')\, V(q)\, \delta(p - p_2 - q)\, \delta(p_2 + q - p')\, d^4p_2\, d^4q =$$

$=$ 

external lines have same p

Integrating by p_2

$$G^{(1)}(p, p') = G^{(1)}(p)\, \delta(p - p')\, (2\pi)^4$$

$$iG^{(1)}(p) = -G^0(p) \int \frac{d^4q}{(2\pi)^4}\, G^0(p - q)\, V(q)\, G^0(p)$$

This expression allows qualitative interpretation of diagrams. We can imagine that a particle with momentum p emits interaction quantum q and acquires momentum $p - q$. After that particle absorbs the quantum back.

$\Longrightarrow$ $iG(k) \equiv i\, G_{\alpha\alpha'}(k, \omega)$ Causal GF

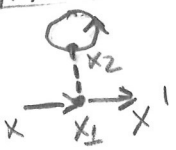
$\longrightarrow$ $iG^0(k) \equiv i\, G^0(k, \omega)\, \delta_{\alpha\alpha'}$ Unperturbed GF

$\bullet - - - \bullet$ $-iV(q)$ Fourier transform of interaction

$\circlearrowleft$ $n_0 = -i \int \frac{d^4p}{(2\pi)^4}\, G(p)\, e^{i\omega t}$, $t \to +0$ Unperturbed electron density

1. The integration over all intermediate momenta and frequencies $dk/(2\pi)^4$ and summation over dummy spin indices is implied taking into account momentum and energy conservation laws.
2. Multiply the result by $(-1)^F$, where F - number of loops without bubbles

Likewise we can calculate another diagram

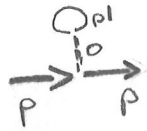


$$= -\int G^0(x - x_1)\, G^0(x_1 - x')\, G^0(t = 0, t = -0)\, V(x_1 - x_2)\, d^4x_2\, d^4x_1 =$$

$$= -V(q=0)\, in\, (2\pi)^{-8} \int G^0(p_1)\, G^0(p_2)\, e^{ip_1(x - x_1) + ip_2(x_1 - x')}\, d^4x_1\, d^4p_{1,2} =$$

$n = -iG(t = -0)$

$$= -i\, (2\pi)^{-4}\, n\, V(q=0) \int G^0(p_1)\, G^0(p_2)\, \delta(p_1 - p_2)\, e^{i(p_1 x - p_2 x')}\, d^4p_1\, d^4p_2 =$$

Fourier component $= -i\, n\, V(q=0)\, G^0(p)\, G^0(p) =$ 

since we can represent $n = -i \int \frac{d^4p}{(2\pi)^4}\, G(p)\, e^{i\omega t}$, $t \to +0$

- <u>Electron-phonon interaction</u>

Besides scalar electron-electron interaction, another important interaction in solid state systems is electron-phonon interaction

- E-ph coupling is described by terms $\psi^{+}(x)\,\psi(x)\,\varphi(x)$
- Only even-order terms will contribute the perturbation expression since otherwise unpaired phonon field will give <u>zero vacuum average</u>.

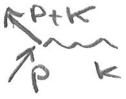
Phonon propagator $\quad i D(x) = \langle T\, \varphi(x)\, \varphi(x') \rangle$ $\qquad$ (9)

$$i D(k) = \frac{i\omega_k^2}{\omega^2 - \omega_k^2 + i0}$$

Feynman rules

$i D(k)$ Exact phonon propagator

$i D^{0}(k)$ Unperturbed phonon propagator



$-ig$ E-ph coupling constant

Any correction to a Green function is easily written down with the aid of these rules. For instance, the expression for Fig. 17 is

$$-i\delta_{\alpha\beta}G^{(0)2}(p)(2\pi)^{-20}\int \mathrm{d}^4q_1\cdots\mathrm{d}^4q_4\,U(q_1)\,U(q_2)\,U(q_3)\,U(q_1+q_2+q_3)\,U(q_4)$$
$$\times G^{(0)}(p-q_1)G^{(0)}(p-q_1-q_2)G^{(0)}(p-q_1-q_2-q_4)G^{(0)}(p-q_1-q_2-q_3-q_4)$$
$$\times G^{(0)}(p-q_4)\int \mathrm{d}^4p_1 G^{(0)}(p_1)G^{(0)}(p_1+q_3)G^{(0)}(p_1+q_2+q_3)\,G^{(0)}(p_1+q_1+q_2+q_3).$$

We next consider the symmetrised version of the diagram technique for two-particle interactions. The symmetrised quantity $\Gamma^{(0)}_{\gamma_1\gamma_2,\gamma_3\gamma_4}(x_1, x_2; x_3, x_4)$ was introduced earlier. By definition, $\Gamma^{(0)}$ depends only on the coor-

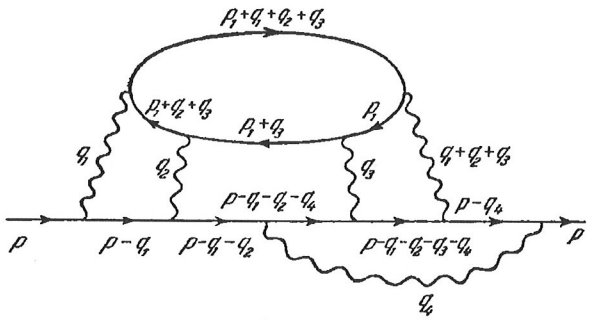


Fig. 17

dinate differences. The Fourier components of $\Gamma^{(0)}$ will therefore contain $\delta(p_1+p_2-p_3-p_4)$. In view of this it will be convenient for us to define the Fourier component of $\Gamma^{(0)}$ at once as

$$(2\pi)^4\delta(p_1+p_2-p_3-p_4)\Gamma^{(0)}(p_1, p_2; p_3, p_1+p_2-p_3)$$
$$=\int \mathrm{d}^4x_1\cdots\mathrm{d}^4x_4\,\Gamma^{(0)}(x_1x_2, x_3x_4)\,e^{-ip_1x_1-ip_2x_2+ip_3x_3+ip_4x_4}.$$

The Fourier transform of the first order term corresponding to the diagram of Fig. 9 is

$$-iG^{(0)2}(p)\int\frac{\mathrm{d}^4p_1}{(2\pi)^4}\Gamma_{\alpha\gamma,\beta\gamma}(pp_1; pp_1)G^{(0)}(p_1).$$

The diagram in momentum space is illustrated in Fig. 18. The general rules for drawing the diagrams are just the same as before. In particular,

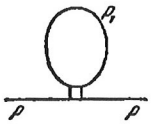


Fig. 18

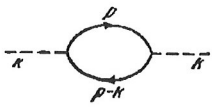


Fig. 19

the coefficient of the nth order diagram only differs from the coefficient in the coordinate form by the factor $(2\pi)^{-4n}$.