Let's consider again the model Hamiltonian

$$\hat{H} = \int \left(-\psi_\alpha^+ \frac{\nabla^2}{2m} \psi_\alpha + \frac{\lambda}{2} \psi_\alpha^+ \psi_\beta^+ \psi_\beta \psi_\alpha \right) d^3 r$$

$$\begin{cases} \psi^M = e^{\hat{H}\tau} \psi e^{-\hat{H}\tau} \\ \bar{\psi}^M = e^{\hat{H}\tau} \psi^+ e^{-\hat{H}\tau} \end{cases}$$

$-[\psi_\alpha^+ \psi_\alpha, \psi_\beta] =$
$\psi_\beta \psi_\alpha^+ \psi_\alpha - \psi_\alpha^+ \psi_\alpha \psi_\beta$
$= \delta_{\alpha\beta} \psi_\alpha$

$-[\psi_\alpha^+ \psi_\alpha, \psi_\beta^+] =$
$\psi_\beta^+ \psi_\alpha^+ \psi_\alpha - \psi_\alpha^+ \underbrace{\psi_\alpha \psi_\beta^+} = -\psi_\alpha^+ \delta_{\alpha\beta}$
$-\psi_\beta^+ \psi_\alpha^+ \delta_{\alpha\beta}$

Use Eqs. of motion

$$\frac{\partial}{\partial \tau} \psi^M = [H, \psi^M] \qquad \frac{\partial}{\partial \tau} \bar{\psi}^M = [H, \bar{\psi}^M]$$

$$\begin{cases} \left(\frac{\partial}{\partial \tau} - \frac{\nabla^2}{2m}\right) \psi_\alpha + \lambda \bar{\psi}_\gamma^* \psi_\gamma \psi_\alpha = 0 \\ \left(\frac{\partial}{\partial \tau} + \frac{\nabla^2}{2m}\right) \bar{\psi}_\alpha - \lambda \bar{\psi}_\alpha^* \bar{\psi}_\gamma^* \psi_\gamma = 0 \end{cases}$$

$$G^M = \mathrm{Tr}\left(\rho T \psi^M(r_1, \tau) \bar{\psi}^M(r_2, 0)\right) = \langle T \psi^M(r_1, \tau) \bar{\psi}^M(r_2, 0) \rangle$$

$\left(\frac{\partial}{\partial \tau} - \frac{\nabla^2}{2m}\right) G^M$

$$\frac{\partial}{\partial \tau} G^M_{\alpha\beta} = \delta(r_1 - r_2)\delta(\tau) + \mathrm{Tr}\left(\rho T \frac{\partial}{\partial \tau} \psi^M_\alpha(r_1, \tau) \bar{\psi}^M_\beta(r_2, 0)\right) =$$

$$= \delta(r_1 - r_2)\delta(\tau) + \mathrm{Tr}\left(\rho T \left(\frac{\nabla^2}{2m} \psi_\alpha - \lambda \bar{\psi}_\gamma^* \psi_\gamma \psi_\alpha\right)_{(r_1,\tau)} \bar{\psi}_\beta^M(r_2, 0)\right) =$$

Therefore
$$\left(\frac{\partial}{\partial \tau} - \frac{\nabla_1^2}{2m}\right) G_{\alpha\beta}(r_1, \tau, r_2, 0) = \delta(r_1 - r_2)\delta(\tau)\delta_{\alpha\beta} - \lambda \mathrm{Tr}\left(\rho T \left(\bar{\psi}_\gamma^+ \psi_\gamma \psi_\alpha\right)(r_1, \tau) \bar{\psi}_\beta^M(r_2, 0)\right)$$

$$= \cancel{\delta(r_1 - r_2)} \delta(\ \ \delta(x_1 - x_2) - \lambda \mathrm{Tr}\, \rho T\left(\cancel{\psi_\gamma^+} \cancel{\psi_\gamma} \psi_\alpha{}_{(r_1,\tau)} \bar{\psi}_\beta{}_{(r_2,0)}\right)$$

Use the same trick with mean-field approximation

$$\langle T (\bar{\psi}_\gamma \psi_\gamma \psi_\alpha (x_1) \bar{\psi}_\beta (x_2)) \rangle = \langle T \psi_\gamma^+{}_{(x_1)} \psi_\beta^+ (x_2) \rangle \langle \psi_\gamma \psi_\alpha (x_1) \rangle =$$

$$= F^+(x_1, x_2)\, \Delta(x_1)$$

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