Stochastic Processes 2019

Some general rules for the examination

- Please justify each step.
- If you refer to a statement of the course, then please quote the statement exactly.
- I will ask for basic definitions and the statements of the main theorems as well.

Topics

(1) **Basic definitions**

- What is a probability space?
- What is a random variable?
- Definition of independence of events and independence of random variables.
- Definition of the expected value $\mathbb{E}\xi$ of a random variable on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ (see script "Introduction into Probability Theory").
- Definition of the p-th moment of a random variable $\xi : \Omega \to \mathbb{R}$.
- Lemma of Borel-Cantelli.

(2) **zero-one laws**

- Formulation and understanding of Zero-one law of Kolmogorov (Proposition 2.1.6).
- Proof of Corollary 2.1.8 and interpretation.
- Corollary 2.1.10 and idea of the proof.

(3) **Sums of independent random variables**

- Importance of Corollary 2.1.8.
- Formulation of the 2-Series Theorem (Proposition 2.2.1).
- Proof of the 'if' part of Example 2.2.2.
- Formulation of the 3-Series Theorem (Corollary 2.2.3) and proof of $(iii) \Rightarrow (i)$.

(4) **Basic limit theorems**

- Formulation of the strong law of large numbers (Proposition 2.2.8).

(5) **Conditional expectations**

- Lebesgue spaces $\mathcal{L}_p$ (Definition 3.1.1 (i) and basic properties (Proposition 3.1.3).
- definition of conditional expectations (Proposition 3.1.6 and Definition 3.1.7).
- Proposition 3.1.5 with interpretation as conditional expectation.
- Proposition 3.1.9.

(6) **Martingales**

- Definition of a martingale (Definition 3.2.2).
- Proposition 3.2.3 and 3.2.4 with proofs (exact usage of Proposition 3.1.8).
- Elementary properties of martingales (Proposition 3.3.1).
- Proof of Proposition 1.2.1 (i).

(7) **Main Theorems about Martingales**

- Uniformly integrable martingales: Formulation and understanding of Theorem 3.8.6 and proof of Proposition 1.2.1 (ii, iii).
- Optional stopping theorem: Definition of a stopping time (Definition 3.4.1), Proposition 3.4.6 (ii), (iii), (iv), and Proposition 3.6.3.